

71. *Alferov O.I., Doctor of Engineering, Professor, Ukrainian State University of Railway Transport*

DISTRIBUTION OF THE NUMBER OF LOADINGS TO FAILURE OF MACHINE ELEMENTS

At a deterministic constant limit level of bearing capacity of the element, an integer number of random in magnitude loadings before sudden failure has a discrete one-parameter geometric distribution with a probability function of the following form

$$Q_m = R_{m-1} - R_m = Q_1(1 - Q_1)^{m-1} \quad (1)$$

where R_m - probability of failure-free operation; m - number of extreme loadings.

However, if the load carrying capacity of each element is a random variable fixed in time, the distribution of the number of loadings to failure is not geometric under repeated random loading. This is evidenced by the expression for the probability of failure-free operation (2), obtained under the assumption that the load and load-carrying capacity are random and have a Weibull distribution or obey the Fréchet law [1, 2].

$$R_m = \prod_{i=1}^m \frac{i\bar{K}^b}{i\bar{K}^b + 1} \quad (2)$$

where, \bar{K} - safety factor calculated from the average values of bearing capacity and load; b – distribution scale parameter.

It is also assumed that bearing capacity and load are independent but similar random variables, i.e. they have the same distribution law and the same coefficients of variation.

It makes sense to consider the distinguishing features of the geometric distribution from discrete distributions of the number of loadings to failure, which correspond to the cases of distributions of similar values of load and bearing capacity according to the Weibull and Fréchet laws.

If there is a Weibull distribution, the probability of failure at the first loading is as follows

$$Q_1 = 1 - R_1 = \frac{1}{\bar{K}^b + 1}, \quad (3)$$

from where $\bar{K}^b = \frac{1 - Q_1}{Q_1}$.

Substituting this value into (2) gives an expression for the probability of failure-free operation under t -fold loading in the form of

$$R_m = \frac{(1-Q_1)^m m!}{\prod_{i=1}^m [(1-Q_1)i + Q_1]}; \quad m = 1, 2, \dots \quad (4)$$

Using (4) we can obtain the probability function of the considered discrete distribution depending on only one parameter Q_1 :

$$Q_m = R_{m-1} - R_m = \frac{Q_1(1-Q_1)^{m-1}(m-1)!}{\prod_{i=1}^m [(1-Q_1)i + Q_1]}; \quad m = 1, 2, \dots \quad (5)$$

This function determines the probability of failure specifically at a given extreme loading and is a discrete analogue of the distribution density of the number of loadings to failure.

The discrete distribution (5) is not found in known references [4] and in [3] this distribution is called hypogeometric.

The probability function of the hypogeometric distribution (5) differs from the corresponding characteristic of the geometric distribution [1, 2, 4] by the presence of a multiplier $\frac{(m-1)!}{\prod_{i=1}^m [(1-Q_1)i + Q_1]}$, which at

small $m > 1$ has a value less than unity, but with increasing m begins to exceed one and increase the probability values Q_m compared to the geometric distribution.

An important characteristic of the hypogeometric distribution is the conditional distribution function (risk function), which is defined as a ratio of

$$\lambda_m = \frac{Q_m}{R_{m-1}} = \frac{Q_1}{(1-Q_1)m + Q_1}; \quad m = 1, 2, \dots \quad (6)$$

This function allows for each successive t -th loading to determine the conditional probability of failure of the object that, having withstood the preliminary $m-1$ loading, did not fail. As follows from (6), the risk function of the hypogeometric distribution is a monotonically decreasing function of the number of loadings m and thus fundamentally differs from the geometric distribution, which has λ_m constant Q_1 . Reduction of the risk of failures due to preloading in a number of cases justifies the expediency of using such a method of ensuring the reliability of elements having random dispersion of bearing capacity.

Consequently, contrary to the previously accepted approach [4, 5], the use of geometric distribution cannot be considered reasonable when predicting mechanical reliability by sudden failures under conditions of random dispersion of the bearing capacity of elements.

Reference:

1. Alferov, A. I. Fundamentals of forecasting and reliability management in conditions of extreme loads : a monograph / A. I. Alferov, A. S. Grinchenko. - Kharkiv: Planeta-Print, 2017. - 135 p.
2. O. Grynchenko, O. Alfyorov. Mechanical Reliability. Prediction and Management Under Extreme Load Conditions. Springer Nature Switzerland AG 2020.
3. Grinchenko A. S. Some applied models of strength reliability at sudden failures. Bulletin of the National Technical University "KHPI". Thematic issue: Dynamics and Strength of Machines, Kharkov, 2003. № 12. T. 1. C. 51-58.
4. Johnson N. L., Kotz S., Kemp A. V. One-Dimensional Discrete Distributions.: Binom. 2012. 559 c.
5. Upton G., Cook I. Oxford dictionary of Statistics. 2008. 453 p.

МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ
ЖИТОМИРСЬКИЙ АГРОТЕХНІЧНИЙ ФАХОВИЙ КОЛЕДЖ



ЗБІРНИК ТЕЗ

XI Міжнародної науково-практичної конференції
**«Перспективи і тенденції розвитку конструкцій
та технічного сервісу сільськогосподарських машин і знарядь»**

<https://doi.org/10.64165/proceeding-ptdstsamt.2025>



11 квітня 2025 року
м. Житомир

<https://doi.org/10.64165/proceeding-ptdstsamt.2025>

УДК 631.2:621.017:615.281:340(477)

Збірник тез доповідей XI Міжнародної науково-практичної конференції «Перспективи і тенденції розвитку конструкцій та технічного сервісу сільськогосподарських машин і знарядь. PTDSTSAMT-2025» з нагоди 30-річчя започаткування підготовки ОС «Бакалавр» за спеціальністю «Агроінженерія». 11 квітня 2025 року. МОН України. Житомирський агротехнічний фаховий коледж. Житомир. 2025. 333 с. <https://doi.org/10.64165/proceeding-ptdstsamt.2025>.

Рекомендовано до друку методичною радою Житомирського агротехнічного фахового коледжу МОН України (протокол від 10.04.2025 р. № 6)

Proceedings of the XI International Scientific and Practical Conference "Prospects and Trends in Development of Structures and Technical Service of Agricultural Machinery and Tools. PTDSTSAMT-2025." on occasion of the 30th anniversary of the initiation of the preparation of the Bachelor's Entity in the specialty "AgroEngineering". April 11, 2025. Ministry of Education and Science of Ukraine. Zhytomyr Agrotechnical Professional College. Zhytomyr. 2025. 333 p. <https://doi.org/10.64165/proceeding-ptdstsamt.2025>.

В збірнику представлені тези доповідей науково-педагогічних працівників, наукових співробітників, аспірантів та студентів Житомирського агротехнічного фахового коледжу, провідних вітчизняних і закордонних закладів вищої освіти та наукових установ, в яких розглядаються завершені етапи розробок.

The collection presents abstracts of reports by scientific and pedagogical workers, researchers, postgraduates and students of the Zhytomyr Agrotechnical Professional College, leading domestic and foreign higher educational institutions and scientific institutions, which consider the completed stages of development.

Передрук або інше відтворення в будь-якій формі в цілому або частково матеріалів, опублікованих у цьому віданні, дозволено лише за посиланням на джерело і дотриманням вимог законодавства