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## ECONOMIC-MATHEMATICAL MODEL OF OPTIMIZATION OF EXPENSES INVESTMENT-BUILDING PROJECT PAYMENT AUTOMOBILE ROADS

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**Annotation.** The multivariance of the methods of developing investment and construction projects leads to the need to analyze existing and construct new economic and mathematical models that take into account the likelihood of the implementation of certain options, and also take into account the conditions of uncertainty and risk. In the field of motor transport infrastructure, investment and construction projects are always large-scale and costly, because their implementation affects different components of socio-economic development of territories. Investment-building projects of toll roads, in general, are difficulty managed systems that change their configuration in the course of multi-step processes. The ability to manage such a system causes the problem of choosing one or another action and inevitably leads to the task of finding the optimal solution in terms of management.

In the article an algorithm of the economic-mathematical model of optimization of expenses of investment-construction project of a paid motorway in the conditions of uncertainty and risk is developed. The description of the system's work is based on the "Markovian" decision-making processes, and optimization of expenses was carried out by methods of discrete and dynamic programming, in accordance with the general concept of analysis and optimization of multi-step tasks.

The proposed model can be a tool in the operational management of the process of developing an investment and construction project, on the basis of which will be further developed the schedules and calendar plan for its implementation.

**Key words:** costs, dynamic programming, investment-construction project, economic-mathematical model, paid motorway, risk.

### Introduction

To date, the multivariate ness of the options for the development of investment and construction projects leads to the need for an analysis of existing and building new economic and mathematical models that take into account the likelihood of the implementation of certain

options for project development, and also take into account the conditions of uncertainty and risk.

### Formulation of problem

Taking into account the adoption of the law aimed at improving the legislative regulation of the construction and operation of public roads of state importance on the terms of concession in the near future (in 2019) there will arise a need to develop and implement investment-construction projects for toll roads, hence the need to develop new and improve existing decision-making models as a result of their implementation.

### Analysis of recent research results

Issues of investing in transport infrastructure objects, in particular regarding financing of socially important projects in this area, were studied by Ukrainian and foreign scholars Y. Bastia [6], T. A. Vorkut [1], I. V. Heyets [4], P. Drucker [7], K. Eigenram [8].

The purpose of research. Development of algorithm of economical-mathematical model of optimization of expenses of investment and construction project toll road with using "Markov" processes of decision making and methods of discrete and dynamic programming.

### Results of research

On February 27, 2018, the Verkhovna Rada of Ukraine amended the law "Concessions", "Concessions for the construction and operation of highways", "About auto roads", "On sources of financing of road economy of Ukraine", "On the alienation of land plots of other objects of real estate, which are located on them, which are in private ownership, for public needs or from the motives of social necessity." Adoption of certain legislative norms opens up new opportunities for attracting capital, in particular private, for the development of modern motor transport infrastructure of Ukraine.

The practice of attracting private investment for the construction of concession roads is not new. Thus, in such developed countries as France and Italy, financing of road infrastructure development is mainly done through public-private partnership. For example, in France, at the expense of private investment, a network of highways with a length of about 7 thousand km is being built, which are serviced at the expense of usage fees in accordance with established tariffs. In Germany, only certain areas of roads are paying that lead to the recreational areas of the Alpine Mountains are paid. At the same time, on the territory of the EU, the fee for using the network of highways is strictly charged. [3, p. 60]

When considering investment projects in the area of motor transport infrastructure, it should be noted that they are always large-scale and costly; therefore their implementation affects different components (economic, socio-cultural, ecological, industrial and others) socio-economic system of the territory. However, road construction projects are a special case of investment projects, which differ from most projects with such characteristic features:

1. Project product.
2. Number of investors.
3. Level of state regulation.
4. Risks of funding sources.
5. Number of projects.
6. Dependence of the project on the time of work and weather conditions.
7. Strategic constraints on the transfer of the project to private ownership [2, p. 58].

And, the investment and construction project of a toll road, in general, is a complex managed system that changes its configuration in the course of multi-step processes. The ability to manage the system leads to the emergence of a problem of choice of actions and inevitably leads to the task of finding the optimal solution in terms of management. This type of task is a task of dynamic programming, built on, the so-called principle of optimality: According to him, optimal behavior is characterized by the fact that, whatever the initial state and decision at the initial moment, the following solutions should be an optimal behavior relative to the state obtained as a result of the first decision.

The method of dynamic programming is that optimal management is built gradually. At each step, only this step is optimized. At the same time, at each step the management is selected in the light of the consequences, since management, optimizing the target function only for this step, can lead to a non-optimal effect of the entire process. Management at every step should be optimal in terms of the process as a whole. Whatever the initial state of the system before the next step, the management at this stage is chosen so that the gain at this step plus the optimal gain at all subsequent steps was maximal.

The general optimization task can be described as a model of dynamic programming in the following conditions:

1. The task can be interpreted as a n-step process of management, and the overall performance can be presented as the sum of performance indicators at each step;

2. The structure of a task must be defined for any number of steps  $n$  and not depend on that number.

3. At each step, the state of the system is determined by the finite number of "m" state parameters and controlled by the finite number of "r" variables, with  $m$  and "r" not depending on the number of steps "n".

4. The choice of "k" step does not affect the previous steps, and the state at the beginning of this step is the function of only the previous step and the control selected on it.

Dynamic programming processes can take different forms. We propose an analytical model of the process of making a well-founded solution, which will be both fairly general and computational. This model is conditionally stationary because it assumes that the investment-construction project of a toll road is executed for a short period of time. To describe the work of the system, Markov processes of decision-making are used, and for optimization – methods of discrete and dynamic programming, in accordance with the general concept of analysis and optimization of multi-step tasks.

Markov problem of decision, described in R. Howard's work [5], is the task of mathematical programming, which is applicable to multi-step decision-making tasks at risk. In them, the process of changing the state of any system is that at random moments of time  $t_0, t_1, t_2, \dots, t_k$  the system is manifested in one or another prior known discrete state in a consistent manner.

This random sequence of events is called the Markov chain, if for each step the probability of transition from one state "S<sub>i</sub>" in any other "S<sub>j</sub>" does not depend on when and how the system went into "S<sub>i</sub>".

The Markov chain is described with the help of probability states, and they form a complete group of events, so their sum is equal to one.

Consequently, an investment and construction project of a toll road will be a system that at any fixed time can be in one of the numerical states (stage of project execution), which is numbered as  $E_j = 1, 2, \dots, N$  and assume that at discrete moments of time  $t = 0, 1$ , the system moves from one state to another. In addition, processes of state change are not deterministic, but stochastically controlled by a transition matrix:

$$P = (P_{E_i;E_j}),$$

where:  $P_{E_i;E_j}$  the probability of transition of the condition from the stage "i" to the stage "j" of the investment and construction project of the toll road.

We introduce the following functions:  $X_t(E_i)$  – the probability that the system at the time (t) is in the state of  $E_i = 1, 2, \dots, N$ , provided that  $t = 1, 2, \dots$

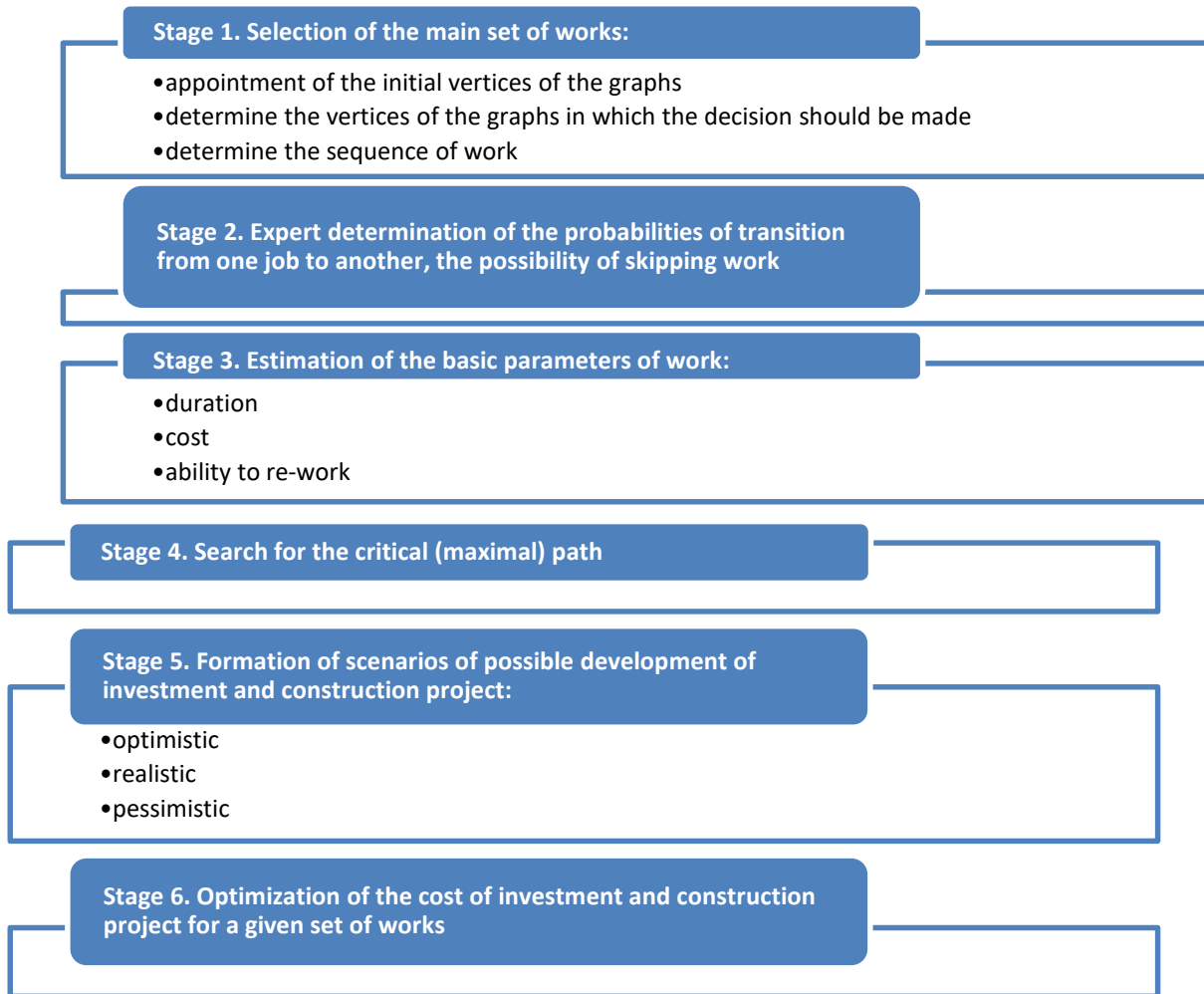
Then, according to the theory of probability:

$$X_{t+1}(E_j) = \sum_{i=1}^N P_{E_i;E_j} \times x_t(E_i), E_j = 1, 2, \dots, N$$

$$x_0(E_i) = C_i.$$

Given that in the theory of Markov processes, the asymptotic behavior of a function is considered  $x_t(E_i)$ , at  $t \rightarrow \infty$  and if all transitional probabilities  $P_{E_i;E_j}$  positive then the definite functions approach the values  $x_t(E_i)$ , satisfying the equation of "stationary regime":

$$X(E_j) = \sum_{i=1}^N P_{E_i;E_j} \times x(E_i), E_j = 1, 2, \dots, N$$



**Fig. 1.** Algorithms for constructing an economical-mathematical model for optimizing the costs of an investment-construction project of a toll road.

*Source:* compiled by the author.

In this model, at each step, one of the sets of such matrices can be selected as a transition matrix, accordingly, we can select a matrix for determining the policy  $\alpha (P_{(\omega)} = (P_{Ei;Ej}(\alpha)))$ .

Next, let's assume that not only the fortune changes at each step, but also the costs associated with the development of the project, which is the function of the initial and final state and decision. In this case, the expression  $R_{(\omega)} = (r_{Ei;Ej}(\alpha))$  is a cost matrix..

The process described above is the Markov process of decision making, therefore, the essence of the problem solution is to choose a sequence of decisions, which minimize the mathematical expectation of the costs incurred in the N-step process, at a given initial state of the system.

Let it  $\alpha_1^1; \alpha_2^1; \alpha_3^1$  – alternative variants of implementation of the investment project in period “I” and  $h_1^1; h_2^1; h_3^1$  – the corresponding expenses for the implementation of the project for these options in period “I”.

Since alternatives to the implementation of the project arise at each of the stages, designate for the II period alternate project implementation and their costs

$\alpha_1^2; \alpha_2^2; \alpha_3^2$  and  $h_1^2; h_2^2; h_3^2$ . Then for the III period, alternative variants of the project implementation and their costs will be  $\alpha_1^3; \alpha_2^3; \alpha_3^3$  and  $h_1^3; h_2^3; h_3^3$ .

The solution to the task of minimizing costs is carried out in stages and begins with the search for a minimum number of costs in the III period. Then such procedures are repeated for the II and I periods. The final decision on the development of the project is formed from a consistent choice, starting with the first period of such alternatives, at which the cost index is minimal.

Uncertainty and risk factors are assessed by experts as possible additional costs at each of the stages of the development of an investment and construction project and determined using non-formalized subjective assessment methods. In some cases, the magnitude of the influence of these factors can be determined on an objective basis (for example, on the basis of statistics on the progress of adverse events on similar objects).

Accordingly, the main task is to build an optimal solution in all three stages of choosing a minimum cost indicator taking into account the component of uncertainty and risk. To do this, the probability of its occurrence and the value of gain (the cost minimization,

taking into account the component of uncertainty and risk) is calculated, which can be obtained taking into account this probability. The calculation is performed for each vector of decisions from the initial node of decision-making to the final node of the corresponding result, with the selection of the branch, which leads to the maximum gain and return to the previous decision node, which assigns this value to the winnings.

Construction of the economic-mathematical model is reduced to the task of managing the process of developing an investment project, the algorithm for solving which consists of the following main stages.

### Conclusions

1. Investment and construction projects in the sphere of motor transport infrastructure are always complex, large-scale and costly. Their realization affects the various components (economic, socio-cultural, ecological, industrial.) of the socio-economic system of the territory and accordingly requires the application of various models of management of them both at the design stage and in the process of implementation.

2. The proposed model may prove to be a powerful tool in the operational management of the project development process, on the basis of which the graphs and the plan for its implementation will be built in the future. Thus, interested parties (public or private partners), using the developed model, are able to quickly make decisions not only before the start of the investment and construction project, but also during its implementation, as well as to receive detailed information about possible losses at the decision points.

### References

1. *Vorkut, T. A.* (2008). *Proektnyj analiz*. Kyiv. UTsDK.
2. *Zalunin, V. F., Schehlova, O. Yu., Lazhe, M. V.* (2013). Estimation of economic efficiency of construction of highways. *Derzhava ta rehiony*. Vol. 2 (71). 55-59.
3. *Zatonats'ka, T. H.* (2013). Ways of formation of modern transport infrastructure in Ukraine. *Finansy Ukrainy*. Vol. 8. 54-61.
4. *Heiets, V. M.* (2010). *New course: reforms in Ukraine. 2010-2015*. Kyiv. NVTs NBUV.
5. *Hovard, R. A.* (1964). *Dynamic Programming and Markov Processes*. Moscow.
6. *Bastiat, F.* (2005). *Sophismes Economique France*: Belles Lettres.
7. *Drucker, P. F.* (1995). *Managing for the Future*—1st ed. Plume.
8. *Eijgenraam, C. J. J., Koopmans, C. C., Prij J., Rosenberg, F. A., Tang, P. J. G., Verster, N.* (2000). OEEI: Evaluatie van infrastructuurprojecten: leidraad voor kostenbatenanalyse, *Tijdschrift Vervoerwetenschap* 31, 28-33.

### Список літератури

1. *Воркут Т. А.* Проектний аналіз. Київ. УЦДК. 2008. 440 с.
2. *Залунін В. Ф., Щеглова О. Ю., Лажє М. В.* Оцінка економічної ефективності будівництва автомобільних доріг. Держава та регіони Серія: Економіка та підприємництво. 2013. № 2 (71). С. 55–59.
3. *Затонацька Т. Г.* Шляхи формування сучасної транспортної інфраструктури в Україні. *Фінанси України*. 2013. № 8. С. 54–61.
4. *Гейця В. М.* Новий курс: реформи в Україні. 2010-2015. Київ. НВЦ НБУВ. 2010. 232 с.
5. *Ховард Р. А.* Динамическое программирование и марковские процессы. Москва. Издательство "Советское радио". 1964. 189 с.
6. *Bastiat F.* *Sophismes Economique*. France. Belles Lettres. 2005. 290 p.
7. *Drucker P. F.* *Managing for the Future*. 1st ed. Plume. 1995. 384 p.
8. *Eijgenraam, C. J. J., Koopmans, C. C., Prij J., Rosenberg, F. A., Tang, P. J. G., Verster, N.* OEEI: Evaluatie van infrastructuurprojecten: leidraad voor kostenbatenanalyse. *Tijdschrift Vervoerwetenschap*. 2000. №31. P. 28–33.

### МОДЕЛЬ ОПТИМІЗАЦІЇ ВИТРАТ ІНВЕСТИЦІЙНО-БУДІВЕЛЬНОГО ПРОЕКТУ ПЛАТНОЇ АВТОМОБІЛЬНОЇ ДОРОГИ

*О. М. Загурський, В. В. Куканов*

**Анотація.** Багатоваріантність методів розробки інвестиційно-будівельних проектів призводить до необхідності аналізу існуючих і побудови нових економіко-математичних моделей, що враховують ймовірності здійснення тих чи інших варіантів, а також враховують умови невизначеності і ризику. У сфері автотранспортної інфраструктури інвестиційно-будівельні проекти завжди масштабні та затратні, тому, що їх реалізація впливає на різні складові соціально-економічного розвитку територій. Інвестиційно-будівельні проекти платних автомобільних доріг, взагалі являються складно керованими системами, що змінюють свою конфігурацію в ході багатокрокових процесів. Можливість здійснювати управління такою системою зумовлює виникнення проблеми вибору тих чи інших дій і неминуче призводить до задачі пошуку оптимально доцільного рішення з точки зору управління.

В статті розроблено алгоритм економіко-математичної моделі оптимізації витрат інвестиційно-будівельного проекту платної автомобільної дороги в умовах невизначеності та ризику. Опис робіт системи виконано на основі «марковських» процесів прийняття рішень, а оптимізація витрат здійснена методами дискретного та динамічного програмування, відповідно до загальної концепції аналізу і оптимізації багатокрокових завдань.

Запропонована модель може бути інструментом в оперативному управлінні процесом розробки інвестиційно-будівельного проекту, на основі якого в

подальшому будуть побудовані графіки і календарний план його реалізації.

**Ключові слова:** витрати, динамічне програмування, інвестиційно-будівельний проект, економіко-математична модель, платна автомобільна дорога, ризик.

#### МОДЕЛЬ ОПТИМИЗАЦИИ РАСХОДОВ ИНВЕСТИЦИОННО-СТРОИТЕЛЬНОГО ПРОЕКТА ПЛАТНОЙ АВТОМОБИЛЬНОЙ ДОРОГИ

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**Аннотация.** Многовариантность методов разработки инвестиционно-строительных проектов приводит к необходимости анализа существующих и построения новых экономико-математических моделей, учитывающих вероятности осуществления тех или иных вариантов, а также учитывающих условия неопределенности и риска.

В сфере автотранспортной инфраструктуры инвестиционно-строительные проекты всегда масштабные и затратные, поэтому, что их реализация влияет на различные составляющие социально-экономического развития территорий. Инвестиционно-строительные проекты платных автомобильных дорог, вообще являются сложно управляемыми системами, изменяющими свою конфигурацию в ходе многошаговых процессов. Возможность осуществлять управление такой системой обуславливает возникновение проблемы выбора тех или иных действий и неизбежно приводит к задаче поиска оптимально целесообразного решения с точки зрения управления.

В статье разработан алгоритм экономико-математической модели оптимизации затрат инвестиционно-строительного проекта платной автомобильной дороги в условиях неопределенности и риска. Описание работы системы выполнено на основе марковских процессов принятия решений, а оптимизация расходов осуществлена методами дискретного и динамического программирования, согласно общей концепции анализа и оптимизации многошаговых задач.

Предложенная модель может быть инструментом в оперативном управлении процессом разработки инвестиционно-строительного проекта, на основе которого в дальнейшем будут построены графики и календарный план его реализации.

**Ключевые слова:** расходы, динамическое программирование, инвестиционно-строительный проект, экономико-математическая модель, платная автомобильная дорога риск.

