A. KUTSENKO

STRUCTURAL MECHANICS:
The calculations of complex beams and trusses

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In this part of the book the basic questions of Structural Mechanics are consistently expounded, among them: kinematics analysis of complex structures; the calculations of plane trusses are on durability and rigidity of elements of constructions, which work on compression and tension under technique loads and snow loads; a calculation of complex beams is on durability, which are standing on static stage; methods of determination of durability of simple and complex beams under moving loads.

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INTRODUCE

Structural mechanics is one of the basic engineering subjects that anyone involved in construction in practice or theory should understand. Structural mechanics is an analytical continuation of the mechanics of materials and structures (strength of materials).

The main aim of Structural mechanics is for the student to acquire the skills to correctly run the engineering calculations of complex structures. In this edition, the reader will have the opportunity to study the calculation of such complex structures as: beams, trusses, arches and flat frames.

The first part of the manual is presented to the engineering calculation of complex beams and trusses, which are loaded as static and as moving loads.

This book aims to help students taking courses taught in English at National University of life and environmental sciences of Ukraine, Faculty of Design and Engineering, in their studies of one of the most important and most difficult engineering topic. This course will be not taken by foreign students, which is speaking good English and knowing technical and mathematical terms in English, but also Ukrainian students intending to improve their English while studying a professional subject.

A many books are devoted to characterizing the problems, especially in the first part of each chapter. Each section of this textbook consists of two parts: theoretical material and solving typical problems. This book provides examples to illustrate theoretical concepts and show how these concepts can be used in practical situations.

For self-control and better understanding of the theoretical material, almost every paragraph contains the questions for the student's self-control.

CHAPTER I

CALCULATION OF COMPLEX BEAM

THEME 1.
THE QUANTITATIVE STAGE OF KINEMATIC ANALYSIS OF FLAT SYSTEMS

1.1. About kinematics analysis. The classification of flat design models.
1.2. Elements of calculation schemes and their characteristics.
1.3. The degree of freedom of the body and the degree of variability of the system.
1.4. Simple connections.
1.5. Quantitative stage of kinematics analysis.

1.1. About kinematics analysis. The classification of flat design models

The study of the stress-strain state of structures and their elements is performed by calculation models, which are optimally idealized images of the elements of the structure, combined in a certain way into a system of bodies with specific geometric and physical characteristics and loads. The design model, which is a physical model of the structure, should be reflect as accurately as possible the properties of the real object and at the same time should be accessible for solution by the existing design skills and equipment of the constructor.

The computational models could be as a spatial as and a flat. Although for the most part the flat schemes are much simpler, but in many cases, they allow that, the results could be obtained with the required accuracy. There are different kinds of flat complex-element and simple calculation models:

- simple beams and beam systems;
- simple frames and frame systems;
- curved beam and arched systems;
- plate systems;

1.2. Elements of calculation schemes and their characteristics

The first stage of investigations of the stress-strain state of a contracture or building is the kinematics analysis of the design scheme. In addition to assessing the possibility of changing the geometry of the structure (the changing the interposition of its components) the kinematics analysis of the design scheme will allow you to successfully choose the method and rational sequence of the subsequent calculation. At this stage, all elements of the design scheme are considered as rigid bodies. The rigid body means that it does not deform, and load as a component of the design scheme is not used at all. We restrict ourselves to the consideration of flat design schemes, all components of which (elements of structures and loads) belong to one plane.

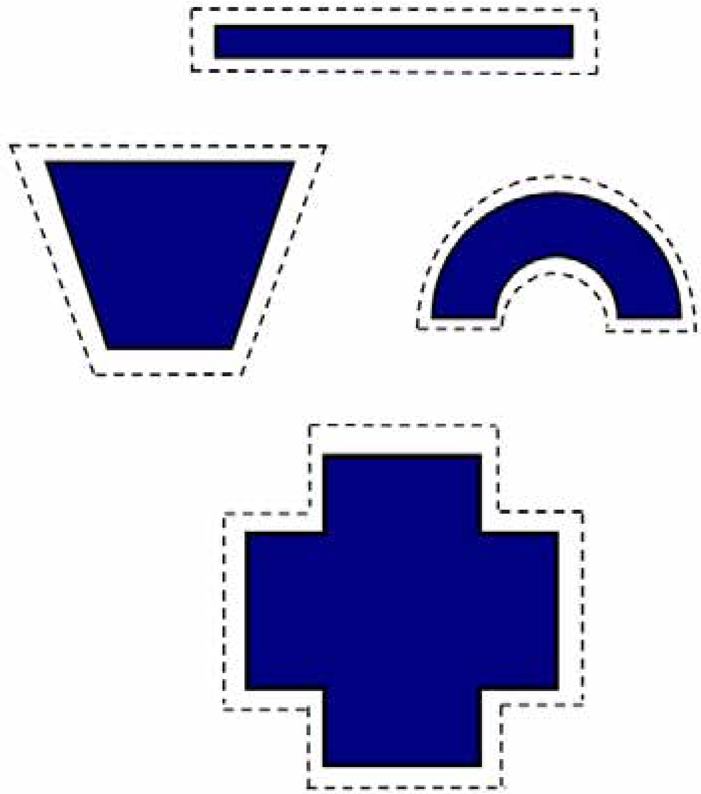


Fig. 1.1. The examples of simple disk.

In construction mechanics, the design schemes are formed from a set of rigid element, which we will name as the simple disks. These simple discs are connected in a closed system by means of idealized connecting devices -supports.

A **simple disk** is the solid bar or plate of the random configuration that does not form any internal isolated contour. Always the simple disk can be made a continuous closed crawl around without crossing it. In the schemes below, the simple disks will be denoted by the letter Д with lower numerical indexes.

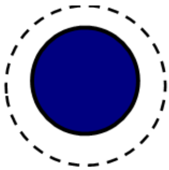


Fig. 1.2. The node

The examples of simple disks are shows on Fig. 1.1. The boundaries of simple disks are showing dashed lines bypassing. In particular, the simple disks are of an arbitrary shape of the plate are perform in Fig. 1,b, d. A simple disk is shape of rectilinear is shown in Fig. 1,a and the simple disk is shape of curved linear is shown in Fig. 1,d.

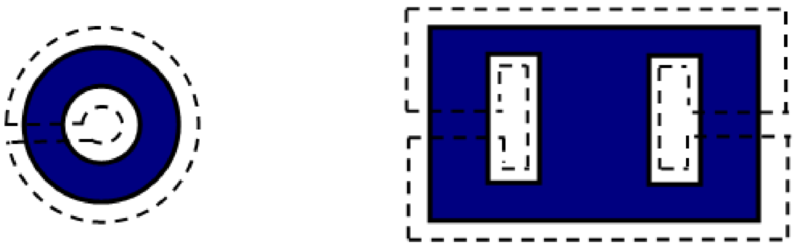


Fig. 1.3. The examples of non-simple disk.

An infinitely small disk, which is transformed into a material point (Fig. 1.2), we will refer to hereafter as a **node**. In the schemes it will denote B

In Fig. 1.3 you could see the depicts a plats which cannot be considered as a simple disks because they have an isolated contour, and, as noted earlier, there is a need to cross at least one groove when circumventing its contours.

1.3. The degree of freedom of the body and the degree of variability of the system

The degree of freedom is the minimum of account of independent geometric parameters that determine the position of a disk or disk system in an arbitrary coordinate system. The position of a finite-sized disk on the plane could fix by three independent generalized coordinates: two of them are linear and one is angular. For example, such coordinates may be, and for the disk Д which is shown in Fig. 1.4.

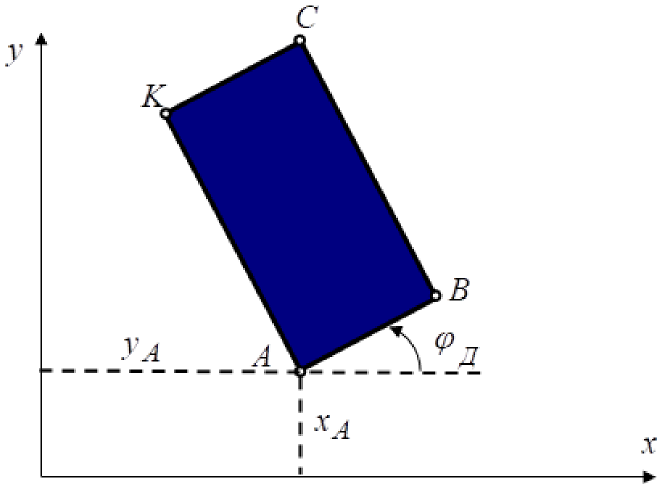


Fig. 1.4. The determination of the degree of freedom for simple disk

For material point B there is no angular coordinate, so the degree of the node is two (Fig. 1.5).

The degree of variability is more important kinematics characteristic for a disk system. The degree of variability is numerically equal to the minimum account of geometric parameters to determine the change in the relative arrangement of the disks, that is, the position of all disks in the coordinate system associated with one of the disks, which is conditionally considered to be stationary and called "ground". The choice of "earth" from the total number of disks of a closed system does not affect the degree of variability of the system, but it will sometimes simplify the course of kinematics analysis. Numerically, the degree of variability of a closed flat system is always three units less than its degree of freedom.

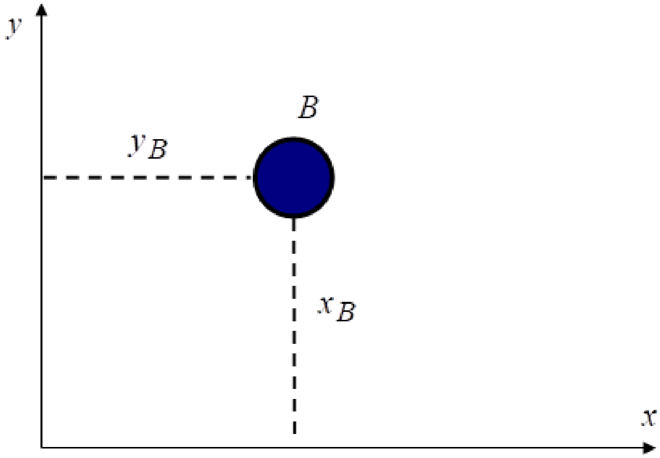


Fig. 1.5. The determination of the degree of freedom for node.

1.4. Simple connections

Let us consider the classification of the main kinds of connection elements, which are used in construction mechanics.

These connection elements are the idealized device models that are used for combining individual disks and nodes with limited object displacement into a system, which is called the complex disk. The connection element reduces the freedom of components and the degree of system variability.

Each of connection element is characterized by kinematic and static properties. Their kinematic properties are manifested in the indisputability of some generalized interplanations of their disks. The number of such displacements is equal to the degree of variability of the system. Static properties of supports are determined by the generalized reactions (forces and moments) that occur in supports between disks in the directions of impossible mutual displacements of disks. The reaction and possible movement are incompatible in idealized connectors in any direction.

Let us just look at some of the idealized devices: simple solder, pin joint and kinematics connection.

A connecting device, which eliminates the mutual linear and angular displacement of the two disks that are connected is called „**simple soldering**" (see Fig. 1.6).

We will refer to this connection simply as a solder. It will denote by letter П with the corresponding index, which is indicating the numbers of the connected disks to it on the design scheme (Fig. 1.6a).

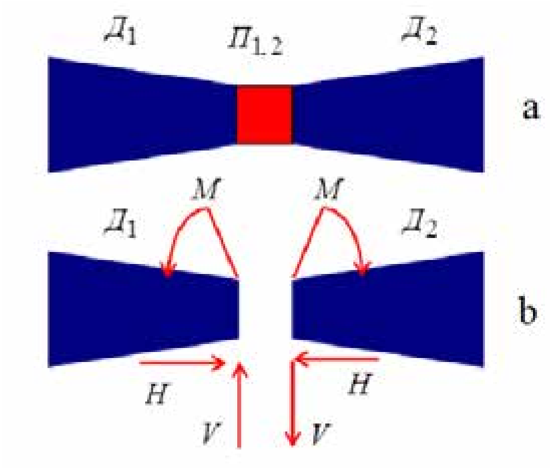


Fig. 1.5. Simple soldering:
a - the design scheme of simple soldering,
b – the reactions of simple soldering.

This connection reduces the number of independent disk drive parameters by three i.e. eliminates three degrees of freedom. The solder reaction is a force which eccentricity line is relatively unknown to the solder center O. This reaction is convenient to present by three components, which have directions to the center of solder O. The two components of it will be forces H and V with directions, which are convenient for study, but with unknown modules. Another component of it is a moment M (Fig. 1.6, b).

The simple cylindrical hinge is called a fixture that connects two disks, shutting off their reciprocal results, while allowing them to rotate relative to an axis which is passing through the center of the hinge (Fig. 1.7a). This type of connection is denoted by the letter Ш. The index of it indicates the quantity of the disks that are joined by it. This connection reduces the number of independent disk motion parameters by two, in that way we can eliminate two degrees of freedom of given structure.

The reaction of a **cylindrical hinge** is a force passing through its geometric center, but in addition to its size (module), it has a previously unknown direction, which must be determined in the subsequent calculation of the construction.

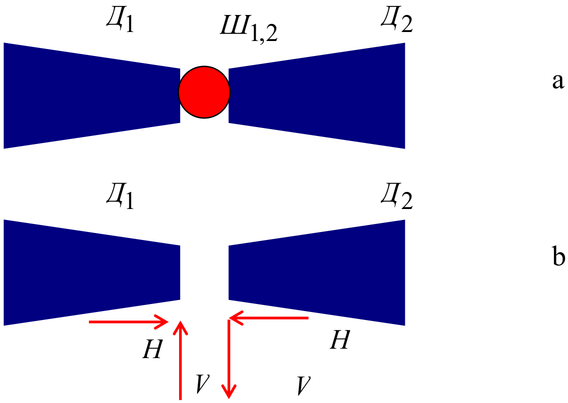


Fig. 1.7. Cylindrical hinge:
a - the design scheme of cylindrical hinge,
b – the reactions of cylindrical hinge.

Usually, in the analytical calculation, the reactive force is replaced by two non-parallel components H and V. They are convenient to choose mutually orthogonal (Fig. 1.7, b).

The bar is connecting the two disks and is eliminating the linear displacements of one disk relative to another one in the direction of the axis which is passing through the points of the hinged joint of the disks is called the **kinematic support** (Fig. 1.8, a).

Such type of support eliminates one degree of freedom and allowing mutual rotation of the discs and their mutual linear movement along the normal to the axis of this connection. The kinematic support has a reaction R arises, the line of action of which runs along the axis of the connection (Fig. 1.8, b).

For convenience, there is a relationship between different type of simple connections, which makes it possible, if necessary, to replace some connections with other one, without changing their kinematic and static properties.

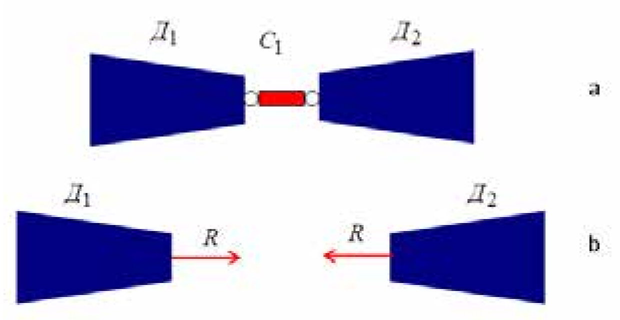


Fig. 1.8. Kinematic support:
a - the design scheme of kinematic support,
b – the reactions of kinematic support.

Thus, a simple cylindrical hinge (Fig. 1.9, a), which eliminates two mutual translational displacements of the discs $Д_1$ and $Д_2$, can be replaced by two kinematic supports C_1 and C_2 (Fig. 1.9, b) that intersect in the center of the hinge they replace. On the contrary, any two kinematic supports C_1 and C_2, which connect a couple of discs $Д_1$ and $Д_2$, can be considered as a single cylindrical hinge $Ш_{1,2}$.

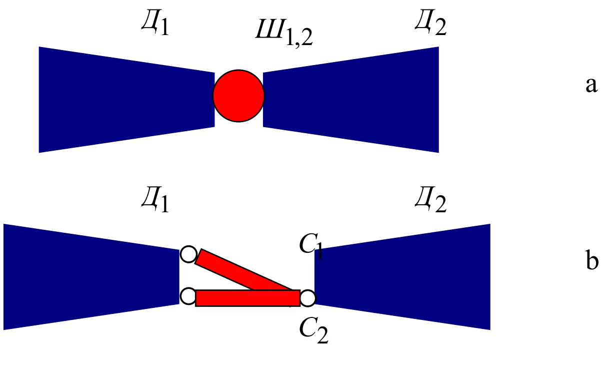


Fig. 1.9. The interchangeability of simple cylindrical hinge by kinematic supports:
a - a simple cylindrical hinge, b - two kinematic supports.

This hinge $Ш_{1,2}$ is located at the point of intersection of the axes of these kinematic supports with the same kinematic and static properties. Such an imaginary hinge is called as a fictitious.

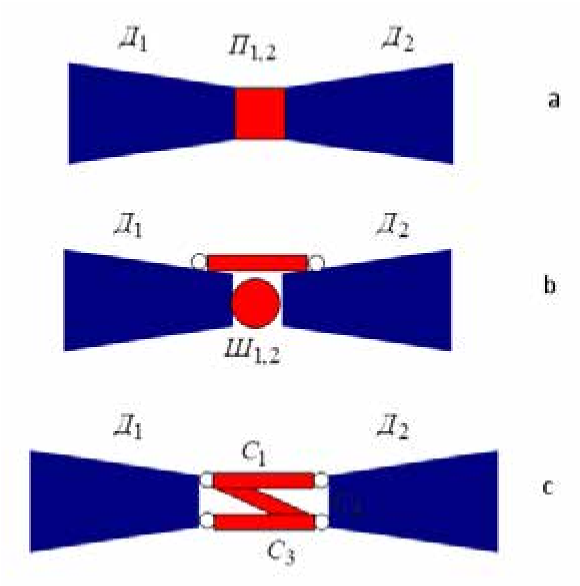


Fig. 1.10. The interchangeability of simple soldering:
a - simple soldering, b - the interchangeability of simple soldering by kinematic support and hinge, c - the interchangeability of simple soldering by three kinematic supports.

Simple soldering $П_{1,2}$ (Fig. 1.10, a) can be replaced by three kinematic supports C_1, C_2 and C_3, the axes of which are not parallel and do not cross at one point (Fig. 1.10, b). Another way we can replace simple soldering $П_{1,2}$ (Fig. 1.10, a) by a hinge and kinematic support that does not pass through the center of the hinge (Fig. 1.10, c).

As the simple joint devices, so the complex hinges and complex soldering are use in the calculation schemes of structures too. A fold hinge is a device, which connects more than two disks. Such a hinge can be divide into simple ones, which joint specific two disks.

It is obvious that the number of simple hinges per unit is less than the number of connected disks. So, for example, a fold hinge jointing four discs is equal to three simple hinges. A complex soldering is determined similarly. If four simple disks joint with the help of one hold soldering, then such soldering is equivalent to four simple soldering.

Therefore, any joint device can be present as a certain number of kinematic supports. Therefore, all devices can be abbreviate as supports, and the forces in such devices can call reactions of supports.

1.5. Quantitative stage of kinematics analysis

As noted early, each calculation scheme of the structure consists of a set of objects. These are disks and points. Each of them has a certain number of degrees of freedom. Connecting devices limit the mutual movements of the components, taking a certain number of degrees of freedom.

When the first stage of kinematic analysis running, the quantitative characteristic of its variability is determined - the degree of variability. To do this, it is necessary the degrees of freedom of the individual participants of the system are summarized. Then the degrees of freedom of all support to subtract from first number of the sum.

Based on the above kinematic properties of the elements of the design schemes, we can write the Chebyshov's formula to determine the degree of geometric variability of the system Г:

$$Г = 3Д + 2В - 3П - 2Ш - С - 3, \quad (1.1)$$

where $Д$ is the number of simple disks, including the "ground" is support disk;

$В$ is the number of material points, that is, joints in which only kinematic supports are connected;

$П$ is the number of simple soldering;

$Ш$ is the number of simple cylindrical hinge;

3 (three) is the number of degrees of freedom of the entire flat computational scheme as one geometrically invariable complex disk in its plane.

Complex hinges and soldering are taken into account as a combination of the corresponding number of simple supports. The degree of geometric variability of the design scheme, calculated by the Chebyshov's formula, characterizes the kinematic properties of the design scheme. If $\Gamma > 0$, then the design scheme of the structure is geometrically variable system.

It means that construction has the ability to change its geometry not only due to the deformation of its components. I this case we have deals with independent possible generalized displacements (velocities) of the elements in such design scheme. Geometrically variable design schemes for calculating structures are used only in special cases (for example, for suspended structures). In most cases, this result means that number of connecting devices is an insufficient and we need to requires a change the method of attack.

Equality $\Gamma = 0$ indicates that the necessary condition for the geometric variability of the design scheme is satisfied. The existing connecting devices in the design scheme is the minimum required number. With the correct location of the connections, it is possible to ensure the immobility of both all the constituent elements and the system as a whole.

In cases when $\Gamma < 0$the system is oversaturated with connections, it means that the geometric invariability of the system can be ensured without some of them (extra supports). However, incorrect placement of connecting devices can lead to local geometric variability of the system. It means that the number of supports between the set of disks of the design scheme is excessive but the rest of the disks are not sufficiently fixed with supports.

Finally, the geometric variability of the design scheme by $\Gamma < 0$ can be verificate only after running qualitative analysis of ones.

Self-control questions

1. *Provide the concept of a simple disk.*

2. Provide the concept of a point.

3. What are kind of connecting elements used in construction mechanics in the design schemes of complex structures?

4. Write down Chebyshev's formula.

5. What is construction called geometrically invariable?

6. What is construction called statically definitely system?

7. How many reactions does the "soldering" has?

8. What is the direction of reaction of kinematic support?

9. How we can replace a simple soldering by another type of connections?

THEME 2.
THE STRUCTURAL ANALYSIS OF COMPLEX STRUCTURES

2.1. The methods of joint the elements of complex structures.
2.2. The content of the structural analysis.

2.1. The methods of joint the elements of complex structures.

Qualitative analysis of the design scheme consists in determining the sequence and methods of forming a system of constituent elements. The connection of the total system or parts of one must be carried out in accordance with the methods of correct combination of elements into geometrically invariable systems, which are complex disks.

In this paragraph, we will consider the main ways of forming the simplest geometrically invariable flat systems using a minimum number of connecting devices. Every presentation of corresponding scheme for combining simple disks into one invariable disk we describe by imaginal "formulas". These formulas are in the form of a fraction. The numerator of this fraction shows a set of elements, which are connected, and the denominator contains a set of connect devices. The name of the new complex invariable disc, which is indicated by a capital Latin letter D with a subscript, is written after the sign "=".

Method of "Dyad". This method consists in the fact that the point support B can be attached to the disk Д using two simple kinematic supports C_1 and C_2, which do not lie on one straight line (Fig. 1.11). This connection method is written by the following formula:

$$\frac{Д + B}{C_1, C_2} \Rightarrow D. \quad (1.2)$$

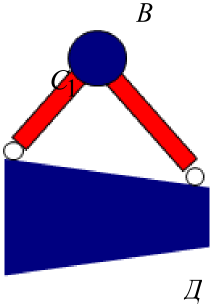


Fig. 1.11. The method of Dyad

In the case when the kinetic supports C_1 and C_2 are located along one straight line, they do not exclude a small possible displacement of the point support B in the direction normal to each of them.

By a small deviation of the point support from the project position, the parallelism of the kinetic supports disturbed, the possible displacements that are allowed by each kinetic supports do not coincide, so the system becomes close to geometrically invariable one. This system is considered as special still. It is called as instantly variable system.

Soldering method. This is the first of three ways to attach one disk to another. According to this method of connection, two disks $Д_1$ and $Д_2$ can be jointed, using soldering $П_{1,2}$ (Fig. 1.12). This fact we can describe by the following formula:

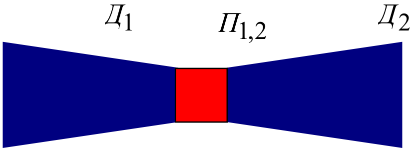
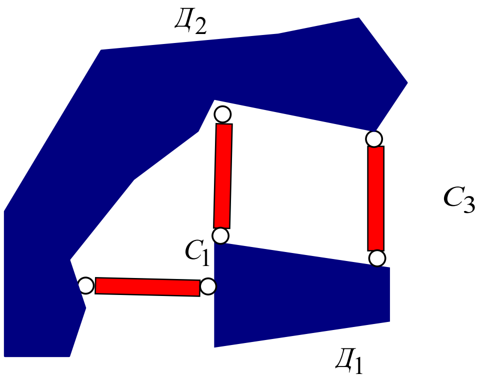


Fig. 1.12. The connection of two simple disk by soldering.

$$\frac{Д_1 + Д_2}{П_{1,2}} \Rightarrow D. \qquad (1.3)$$

Such type of connection is obvious. In general in many cases both disks $Д_1$ and $Д_2$ are considered as one simple disk D.



1.13. The correct connection by Shukhov's method.

Shukhov's method. This is the second way of connecting two disks $Д_1$ and $Д_2$ by three kinematic supports C_1, C_2 and C_3, whose axes do not cross at one point and are not parallel to each other (Fig. 1.13), that is:

$$\frac{Д_1 + Д_2}{C_1, C_2, C_3} \Rightarrow D. \qquad (1.4)$$

Comments. If the axes of the three kinetic supports C_1, C_2 and C_3, cross at point O simultaneously (Fig. 1.14 a), then each of the supports will not eliminate the small displacement of the points of their attachment to the disk in the directions normal of the bar. The disk $Д_1$ will rotate around the instantaneous center of speed O and as a result, the system will be instantaneously variable.

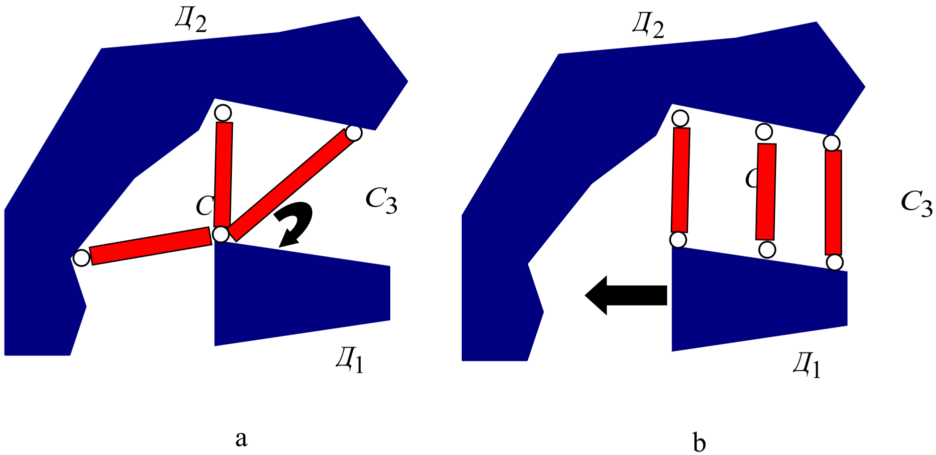


Fig. 1.14. Two cases of incorrect connection by Shukhov's method:
a – the case, when the axes of the three kinetic supports cross at one point,
b - the case, when all axes of the three bars are paralleled.

Another special case, when all axes of the three bars are paralleled (Fig. 1.14 b). It allows for an instantaneous or constant translational displacement of first disk relative to the second one.

Polonso's method. The third way to connect two discs $Д_1$ and $Д_2$ by hinge $Ш_{1,2}$ and a kinematic support C. The axis of kinematic support C does not pass through the center of the hinge $Ш_{1,2}$ (Fig. 1.15):

$$\frac{Д_1 + Д_2}{Ш_{1,2}, C_1} \Rightarrow D. \qquad (1.5)$$

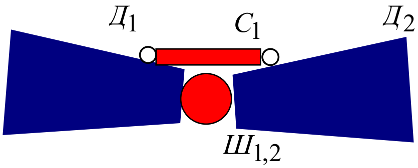


Fig. 1.15. Connection by Polonso's method.

Comments. If the hinge Ш lies on the axis of the support C, then the system is instantly variable.

The method of a hinged triangle. This method consists in connecting three discs using three hinges. Thus, three discs $Д_1$, $Д_2$ and $Д_3$ can be connected using three hinges $Ш_{1,2}$, $Ш_{2,3}$ and $Ш_{3,1}$, which do not lie on one straight line (Fig. 1.16):

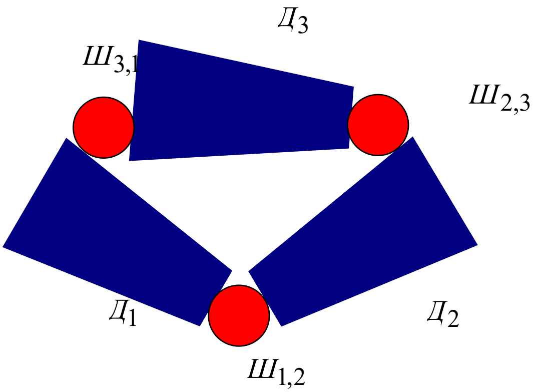


Fig. 1.16. Connection of three disks by method of a hinge triangle.

Comments. If three hinges $Ш_{1,2}$, $Ш_{2,3}$ and $Ш_{3,1}$ lie on the one straight line or all of them are infinitely distant, then it is said that a system of three disks $Д_1$, $Д_2$ and $Д_3$ is instantaneous or geometrically variable.

2.2. The content of the structural analysis

The rules of connection are only for two or three disks. It is necessary to determine the sequence of formation for the design schemes, which consist of a big number of objects. A geometric invariable structure should be formed according to the known rules set forth at each stage. Each of this stage of the creating of the system is called as the "floor" of the design scheme of the structure.

Qualitative (structural) analysis consists in the study of the sequence of connection of elements in accordance with the above methods of formation of the simplest geometrically invariable systems. If we established that two or three elements are combined correctly in one of these ways, such a fragment of the system can be considered as a new enlarged disk and used in conjunction with the rest elements to form new geometrically invariable fragments. This process have to continue until the connections of all elements of the design scheme of the structure are analyzed.

In statically indeterminable systems, the case when we have $Г < 0$, in such a construction, relatively extra supports should remain unused. In many cases, a set of relatively extra supports are chosen ambiguously because we have many choices of them.

The conclusion can be about the geometric variability, geometric or instantaneous variability of the design scheme of the structure. If it turns out that extra devices are used to connect the first set of elements, while for another set of them are not enough, the total design scheme is considered as a geometrically variable. If at least one connection is made as an exception to some method corresponding to the instantaneous variability of the fragment, then the design scheme is instantly variable.

Thus, for the geometric invariability of the flat design scheme of the structure, two conditions must be met: the necessary condition is $Г < 0$; a sufficient condition is the correct formation of the system.

Kinematic analysis must be carried out in a short form, performing detailed schemes, using notations and formulas. The finish of analysis of the calculation scheme is mandatory detailed conclusions.

Self-control questions

1. *What methods of connection disks into a single geometrically invariable structure do you know?*

2. What is the contax of the method of connecting "Dyads"?

3. Write down the formula, which describes the content of the "Dyad" method?

4. How do you connect two simple discs using soldering correctly?

5. Write down the formula that describes the "soldering" method.

6. Write down the formula that describes Polonzo's method.

7. Write down the formula that describes Shukhov's method.

8. What do we can run transformation that Shukhov's method be obtained from Polonzo's method?

9. When is it appropriate to use the hinge triangle method?

10. What do we have the caveats when we are using Shukhov's method?

THEME 3.
CALCULATION OF SIMPLE BEAMS

3.1. Calculation of the cantilever beam.
3.2. Calculation of hinged beam

The elements like as beams are widely used in modern building constructions. We can see beams as horizontal structural elements, which is withstanding shear forces, and bending moments (Fig. 1.17).

Fig. 1.17. Using of beams in the building.

In this paragraph, we will consider the examples of simple beam as the simplest structural elements of buildings. Here we will consider kinematic analysis of these systems and we will remind the basic principles of constructing internal force diagrams for them.

As we knew from the course of strength of materials, the simple statically defined beams can present as two main types by the method of support: cantilever and simple support beams.

Firstly, we will calculation of beams of the first type, namely cantilever beams.

3.1. Calculation of the cantilever beam

The external loads are applied to the beam, these are a uniformly distributed load $q = 1{,}5$ N/m, a couple of forces with a moment $M = 0{,}5$ N·m and a concentrated force $F = 0{,}5$ N (Fig. 1.18). The given beam has the following geometric parameters: $\ell = 5{,}5$ m, $a_1 = 1{,}5$ m, $a_2 = 2$ m.

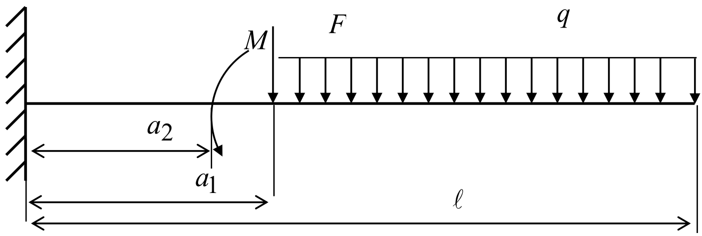


Fig. 1.18. The given cantilever beam.

It is necessary:

1. to run a kinematic analysis for a given beam;
2. to determine the reaction of the beam support;
3. to construct the diagrams of transverse forces and bending moment;
4. From the condition of strength under normal stresses, choose a round and rectangular cross-section of the beam, if $[\sigma] = 1000$ Pa.

1. Kinematic analysis.

1.1. Quantitative stage. The calculation scheme of this design will consist of two disks, one of which is "earth", and one solder (Fig. 1.19).

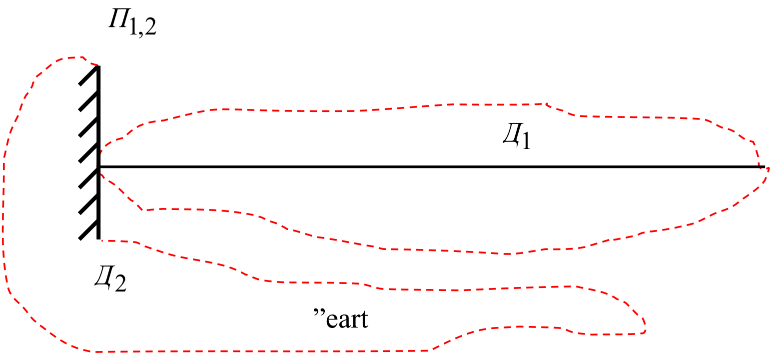


Fig. 1,19. The design scheme of given cantilever beam.

For given construction we have: $Д = 2$, $В = 0$, $П = 1$, $Ш = 0$, $С = 0$. Then according to the Chebyshov formula (1.1), we get:

$$Г = 3 \cdot 2 + 2 \cdot 0 - 3 \cdot 1 - 2 \cdot 0 - 0 - 3 = 6 - 6 = 0.$$

Therefore, the given system is statically determined.

1.2. Quality stage. As already noted earlier, the disks $Д_1$ and $Д_2$ are connected by soldering $П_{1,2}$, and therefore this scheme contains one stage of installation, which is carried out using the soldering method, that is:

$$\frac{Д_1 + Д_2}{П_{1,2}} \Rightarrow D.$$

Conclusion: the given calculation scheme is statically determined and geometrically invariable.

2. Let us determine the reactions of the supports:

$$\Sigma F_{x_i} = 0, \qquad H_A \equiv 0;$$

$$\Sigma F_{y_i} = 0, \quad V_A - q(\ell - a_2) - F = 0,$$
$$V_A = q(\ell - a_2) + F = 5{,}75 \text{ H};$$

$\sum M_{A_i} = 0,\qquad M_A + M - q\dfrac{\ell^2 - a_2^2}{2} - F\cdot a_2 = 0,$

$M_A = -M + q\dfrac{\ell^2 - a_2^2}{2} + F\cdot a_2 \approx 20{,}2$ Н·м.

2.2. Verification: $\sum M_{B_i} = 0$,

$$q\frac{(\ell - a_2)^2}{2} + M_A + M + F(\ell - a_2) - V_A\cdot\ell =$$

$$= 1{,}5\frac{3{,}5^2}{2} + 20{,}2 + 0{,}5 + 0{,}5\cdot 3{,}5 - 5{,}75\cdot 5{,}5 =$$

$$= 31{,}6 - 31{,}6 = 0.$$

1. We build diagrams of transverse forces Q (Fig. 1.20, b) and bending moments M_x (Fig. 120, c):

I portion. $0 \le x_1 \le 1{,}5$ m. $\qquad Q_{x_1} = V_A = 5{,}75$ N,

$M_{x_1} = -M_A + V_A\cdot x_1$, $\qquad M(0) = -20{,}2$ N·m,

$M(1{,}5) = -11{,}6$ N·m;

II portion. $1{,}5$ m $\le x_2 \le 2$ m. $\qquad Q_{x_2} = V_A - F = 5{,}25$ N,

$M_{x_2} = -M_A + V_A\cdot x_2 - M$,

$M(1{,}5) = -12{,}1$ N·m, $\quad M(2) = -9{,}2$ N·m;

III portion. 0 m $\le x_3 \le 3{,}5$ m.

$Q_{x_3} = q\cdot x_3$, $\qquad Q(0) = 0$, $\quad Q(3{,}5) = 5{,}25$ N,

$$M_{x_3} = -q\frac{x_3^2}{2}, \quad M(0) = 0, \; M(3{,}5) = -9{,}2 \text{ N·m}.$$

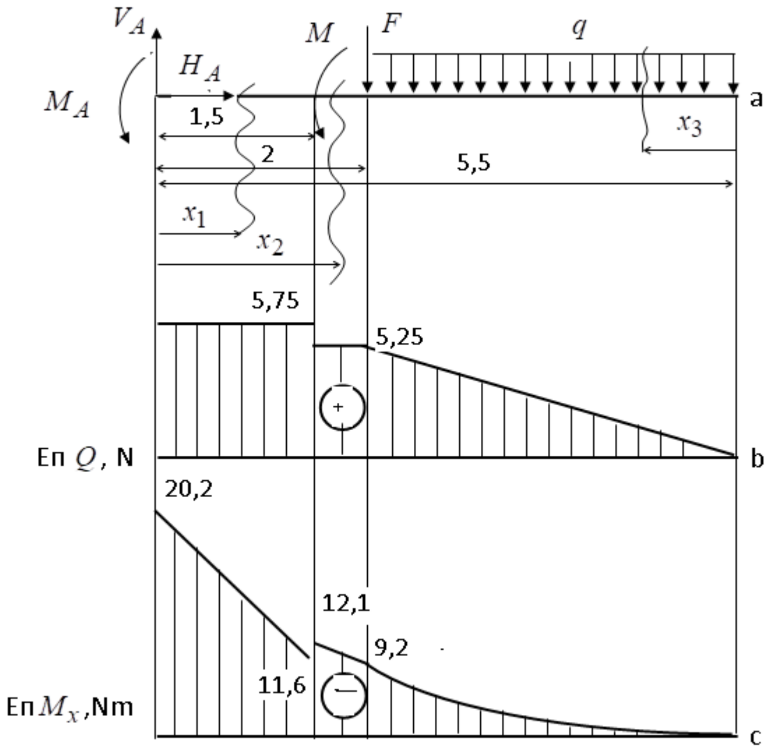


Fig. 1.20. The diagrams of internal efforts of given cantilever beam:
a - given cantilever beam with support reactions,
b – the diagram of shear force, c – the diagram of bend moment.

4. Let us remind that, in the general case, the strength condition for normal stresses by beam bending has the following form:

$$\frac{M_{x_{\max}}}{W_x} \le [\sigma], \tag{1.6}$$

where $M_{x_{\max}}$ is the maximum bending moment, which is determined from the diagram;

W_x is the moment of resistance of the cross-section of the beam above the horizontal axis;

$[\sigma]$ is admissible normal stresses.

Then, from the condition of strength for normal stresses:

a) we select the diameter of a circular cross-section d according to the formula:

$$d \geq \sqrt[3]{\frac{32M_{x_{\max}}}{\pi \cdot [\sigma]}}.$$

In this case, $\left|M_{x_{\max}}\right| = 20{,}2$ N·m.

Then:

$$d \geq \sqrt[3]{\frac{32 \cdot 20{,}2}{3{,}14 \cdot 1000}} \approx 0{,}591\,\text{m} = 59{,}1\,\text{cm}.$$

We accept $d = 60\,\text{cm}$, then $A_K = 0{,}283$ m^2.

b) we select the height h and width b of the rectangular cross-section, if $b = 0{,}4h$, according to the formula:

$$h \geq \sqrt[3]{\frac{6M_{x_{\max}}}{0{,}4 \cdot [\sigma]}}.$$

Then:

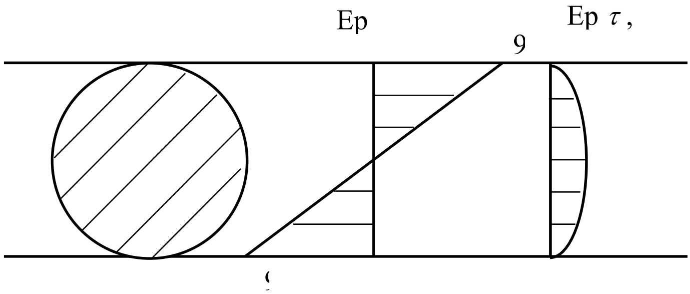
$$d \geq \sqrt[3]{\frac{6 \cdot 20{,}2}{0{,}4 \cdot 1000}} \approx 0{,}675\ \text{m} = 67{,}5\,\text{cm}.$$

We accept $h = 70$ cm, respectively we obtained $b = 28\,\text{cm}$, finally $A_n = 0{,}196$ m^2.

We build diagrams of normal stresses and tangential stresses in dangerous cross-sections.

a) circular section (Fig. 1.21):

$$\sigma_{\max} = \frac{32M_{x_{\max}}}{\pi \cdot d^3} = \frac{32 \cdot 20{,}2}{3{,}14 \cdot 6^3 \cdot 10^{-3}} \approx 0{,}953 \cdot 10^3 = 953\ \text{Pa},$$



$$\tau_{\max} = \frac{4Q_x}{3A_\kappa} = \frac{4 \cdot 5{,}75}{3 \cdot 0{,}283} \approx 27 \text{ Pa.}$$

Fig. 1.21. The diagrams of stresses for circular cross-section:
a – diagram of normal stresses, b – diagram of shear stresses.

b) rectangular section (Fig. 1.22):

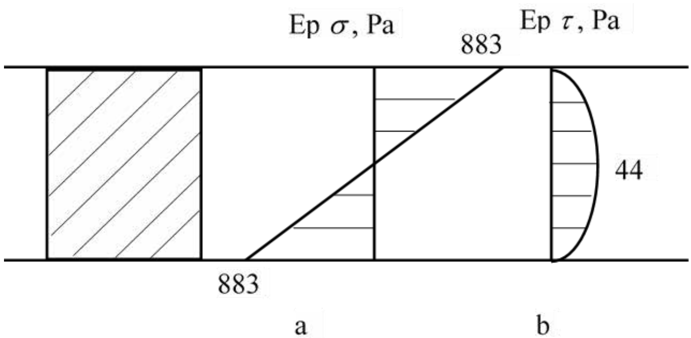


Fig. 1.22. The diagrams of stresses for rectangular cross-section:
a – diagram of normal stresses, b – diagram of shear stresses.

$$\sigma_{\max} = \frac{6M_{x_{\max}}}{0{,}4 \cdot h^3} = \frac{6 \cdot 20{,}2}{0{,}4 \cdot 7^3 \cdot 10^{-3}} \approx 0{,}883 \cdot 10^3 = 883 \text{ Pa,}$$

$$\tau_{\max} = \frac{3Q_x}{2A_{np}} = \frac{3 \cdot 5{,}75}{2 \cdot 0{,}196} \approx 44 \text{ Pa}.$$

The classical description of the strength of materials assumes that the diagram of bending moments is built on compressed fibers. This means that the positive value of the bending moment is placed above the horizontal neutral axis of the plot, and its negative values, respectively, below it. We draw your attention to the fact that in the example we considered, the curve of the bending moment was built on the "tension fibers" of the beam, since in construction mechanics this is a generally accepted rule for constructing this diagram.

According to this rule, the total moment of one-sided forces causes the stretching of the lower fibers, then such bending moment is considered as positive.

The transverse force in any section is calculated as the sum of projections on the vertical axis of all forces acting on the beam on one side of the section. A shearing force that tries to turn the beam clockwise relative to the section is considered positive.

For a better understanding of this rule, let us consider another example of calculating a simple but hinged beam.

3.2. Calculation of hinged beam

Let the beam has the following geometric parameters: $\ell = 5$ m, $a_1 = 2$ m, $a_2 = 1{,}5$ m, $a_3 = 1$ m. According to Fig. 1.23, external load is applied in the form of a pair of forces with a moment $M = 1$ N·m and uniformly distributed load $q = 1$ N/m.

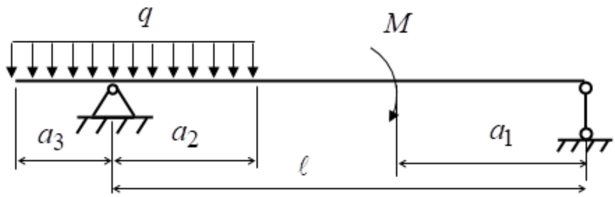


Fig. 1.23. The given simple supported beam.

It is known that the admissible stresses for given beam are equal to $[\sigma] = 16$ kPa.

We should run a kinematic analysis for a given beam and construct the diagrams of the internal efforts of the beam. Select the DoubI T -beam cross-section of the beam from the condition of strength for normal stresses.

1. **Kinematic analysis of the beam**.

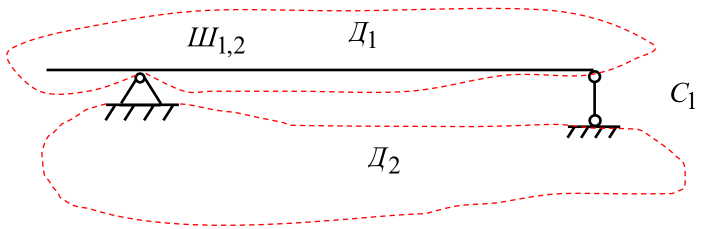


Fig. 1.24. The design scheme of given simple beam.

1.1. **Quantitative stage**. The calculation scheme of this design will consist of two disks, one of which is the "earth", a hinge $Ш_{1,2}$ and a kinematic support C_1 (Fig. 1.24). That is, according to Chebyshov's formula, for a given system we have:

$$Д = 2,\ B = 0,\ П = 0,\ Ш = 1,\ C = 1.$$

Then:

$$Г = 3 \cdot 2 + 0 \cdot 2 - 3 \cdot 0 - 2 \cdot 1 - 1 - 3 = 6 - 6 = 0.$$

Therefore, the given system is statically determined.

1.2. **Quality stage**. Since the two disks are connected to each other by means of a hinge and a kinematic support, it is obvious that the calculation scheme consists of one stage of assembly, which is run by the Polonzo method, i.e.:

$$\frac{Д_1 + Д_2}{Ш_{1,2}, C_1} \Rightarrow D.$$

Conclusion: the given calculation scheme is statically determined and geometrically invariable.

2.1. Let us determine the reactions of the supports:

$$\Sigma F_{x_i} = 0, \qquad H_A \equiv 0;$$

$$\Sigma M_{B_i} = 0, \quad -qa_2\left(\ell - \frac{a_2}{2}\right) - qa_3\left(\ell + \frac{a_3}{2}\right) - M - V_A \cdot \ell = 0,$$

$$V_A = -\frac{qa_2\left(\ell - \frac{a_2}{2}\right) + qa_3\left(\ell + \frac{a_3}{2}\right) + M}{\ell}, \qquad V_A = -2{,}575$$

N;

$$\Sigma M_{A_i} = 0, \qquad R_B \cdot \ell - M + q\frac{a_2^2 - a_3^2}{2} = 0,$$

$$R_B = \frac{M - 0{,}5q\left(a_2^2 - a_3^2\right)}{\ell}, \qquad R_B = 0{,}075$$

N.

2.2.Verification: $\Sigma F_{y_i} = 0$,

$$V_A + R_B + q(a_2 + a_3) = -2{,}575 + 0{,}075 + 1 \cdot 2{,}5 =$$

$$= -2{,}5 + 2{,}5 = 0.$$

3. We build diagrams of shearing forces (Fig. 1.25, b) and bending moments (Fig. 1.25, c):

I portion. $0 \le x_1 \le 1$ m.

$$Q_{x_1} = q \cdot x_1, \qquad Q(0) = 0, \qquad Q(1) = 1 \text{ N},$$

$$M_{x_1} = \frac{q \cdot x_1^2}{2} \qquad M(0) = 0 \text{ , } M(1) = 0{,}5 \text{ N·m};$$

II portion. $1 \text{ m} \le x_2 \le 2{,}5 \text{ m}$.

$$Q_{x_2} = q \cdot x_2 + V_A \text{ ,} \qquad Q(2{,}5) = -0{,}075 \text{ N}, \quad Q(1) = -1{,}575 \text{ N},$$

$$M_{x_2} = \frac{q \cdot x_2^2}{2} + V_A \cdot (x_2 - 1), \; M(2{,}5) = -0{,}74 \text{ N·m};$$

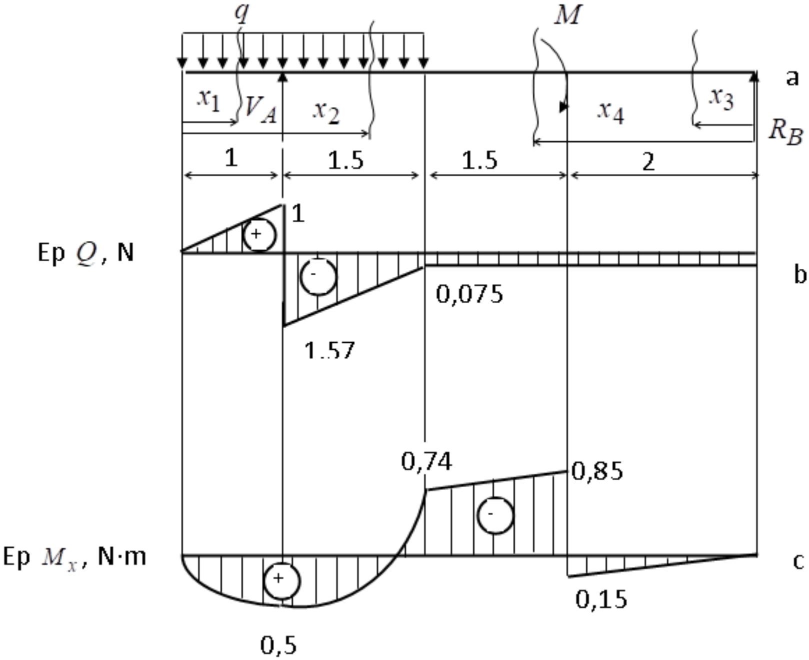


Fig. 1.25. The diagrams of internal efforts of given simple beam:
a - given simple beam with support reactions,
b – the diagram of shear force, c – the diagram of bend moment.

III portion. $0 \text{ m} \le x_3 \le 2 \text{ m}$.

$Q_{x_3} = -R_B = -0{,}075$ N,

$M_{x_3} = R_B \cdot x_3$, $M(0) = 0$, $M(2) = 0{,}15$ N·m.

IV portion. $2 \text{ m} \le x_4 \le 3{,}5 \text{ m}$.

$Q_{x_4} = -R_B = -0{,}075$ N,

$M_{x_4} = R_B \cdot x_4 - M$, $M(2) = -0{,}85$ Н·м, $M(3{,}5) = -0{,}74$ N·m.

4. We select the Double T-beam cross-section from the condition of strength for normal stresses:

$$W_{x_p} = \frac{M_{x_{\max}}}{[\sigma]},$$

in this case $\left|M_{x_{\max}}\right| = 0{,}85$ N·m.

Then:

$$W_{x_p} = \frac{M_{x_{\max}}}{[\sigma]} = \frac{0{,}85}{16 \cdot 10^3} \approx 53{,}1 \cdot 10^{-6} \text{ m}^3 = 53{,}1 \text{ cm}^3.$$

We select Double T-beam № 12, which has the following parameters: $I_x = 350$ cm^4, $W_x = 58{,}4$ cm^3, $S_x = 33{,}7$ cm^3, $h = 12$ cm, $b = 6{,}4$ cm, $d = 0{,}48$ cm, $t = 0{,}73$ cm.

Thus, the maximum normal stresses, which are in the dangerous section, will be equal to:

$$\sigma = \frac{M_{x_{\max}}}{W_{x_p}} = \frac{0{,}85}{58{,}4 \cdot 10^{-6}} \approx 14{,}6 \cdot 10^3 \text{ Pa} = 14{,}6 \text{ kPa} < [\sigma] = 16 \text{ kPa}.$$

With:

$$\sigma_{2,3} = \frac{0{,}85 \cdot 5{,}27 \cdot 10^2}{350 \cdot 10^{-8}} \approx 12{,}8 \cdot 10^3 \text{ Pa} = 12{,}8 \text{ kPa}.$$

We build normal and tangential stress diagrams in dangerous cross-section (Fig. 1.26). For this, we determine the tangential stresses at the points 1, 2, 3 and 4:

$$\tau_1 = 0, \text{ since } S_x = 0;$$

Let us find the first moment of area of the Double T-beam shelf:

$$S_x^{полки} = bt\left(\frac{h-t}{2}\right) \approx 26{,}3 \text{ cm}^3.$$

Then

$$\tau_2 = \frac{Q \cdot S_x^{полки}}{b \cdot I_x} = \frac{0{,}075 \cdot 26{,}3 \cdot 10^{-6}}{6{,}4 \cdot 10^{-2} \cdot 350 \cdot 10^{-8}} \approx 0{,}9 \text{ Pa};$$

$$\tau_3 = \frac{Q \cdot S_x^{полки}}{d \cdot I_x} = \frac{0{,}075 \cdot 26{,}3 \cdot 10^{-6}}{0{,}48 \cdot 10^{-2} \cdot 350 \cdot 10^{-8}} \approx 14{,}1 \text{ Pa};$$

$$\tau_4 = \frac{Q \cdot S_x}{d \cdot I_x} = \frac{0{,}075 \cdot 33{,}7 \cdot 10^{-6}}{0{,}48 \cdot 10^{-2} \cdot 350 \cdot 10^{-8}} \approx 15{,}1 \text{ Pa}.$$

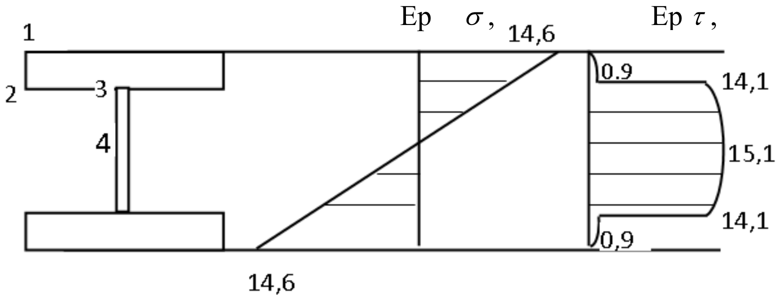


Fig. 1.26. The diagrams of stresses for Double-T cross-section:
a – diagram of normal stresses, b – diagram of shear stresses.

Self-control questions

1. *What types of beams do you know?*
2. *Formulate the principles of construction of the diagram of shearing forces for beams.*
3. *Formulate the principles of constructing the bending moment diagrams for beams.*
4. *How is the kinematic analysis of a beam run?*

THEME 4.
STATIC CALCULATION OF COMPOSITE BEAMS

4.1. Determination of support reactions of a complex beam
4.2. The construction of internal enforces diagrams for complex beams

Over last three years of war in Ukraine, many bridges have been destroyed. Their reconstruction is the task of future generations of Ukrainian builders.

Most bridges in large cities, such as Kyiv, Dnipro and Lviv, can be represented as multi-span beams (Fig. 1.27).

Fig. 1.27. Paton Bridge in Kyiv.

Let us consider examples of the calculation of cantilever - hinged beams, which consist of three or more simple disks.

Statically determined complex beams are called hinged-cantilever.

The quantitative stage of the kinematic analysis of hinged - cantilever beams is run according to Chebyshov's formula. The analysis of the geometric structure consists in the construction of a "floor" or assembly diagram. To do this, each element of a complex beam must be represented as a single-span beam, which has either one support - clamping, or two hinged supports, one of which is hinged-fixed, and the second - hinged-movable. Each of these single-span beams supports either on the base or on other simple beams. A set of simple beams, which rest on each other, creates a scheme of "floors". For this purpose, all intermediate hinges are conditionally replaced with hinged - fixed or hinged - movable supports and trace the support of these beams on each other.

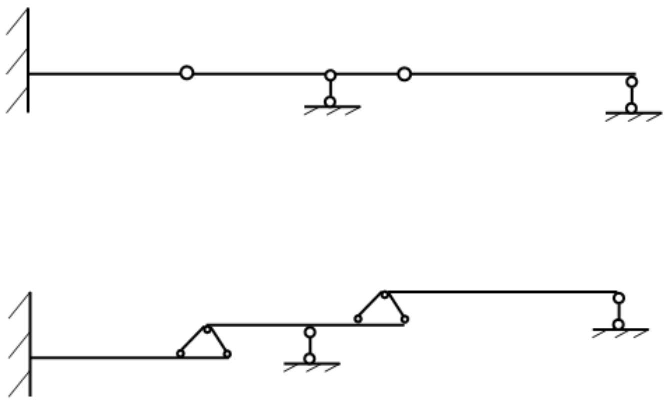


Fig. 1.28. The multi-span beam;
a – the general scheme of three-span beam,
b – the beam floors of installation.

The beam, which is shown in Fig. 1.28, a, consists of three elements. It can be considered as one formed from three single-span beams.

The first two beams directly are supported on the base (Fig. 1.28, b), form the first "floor", the third beam is the second "floor".

The "floor" (assembly) scheme can be used during the static calculation of the hinged-cantilever beam. Thus, the support reactions are determined for the beam of each "floor" separately, and the calculation begins with the beam of the upper "floor". The found support reactions are applied to the beam of the lower "floor" as known external forces, which makes it possible to calculate the reactions of the beam supports of the lower "floor".

Bending moments and shearing forces occur in the cross-sections of the hinged - cantilever beam, which is under the influence of vertical force and moment loads, and the longitudinal forces are known to be zero. The calculation of the beam can be reduced to the calculation of its individual floors: the calculation of support reactions and internal forces can be carried out for the beams of each "floor" separately. The bending moment in any section k is calculated as the algebraic sum of the moments of all forces acting on the beam of the corresponding "floor" on one side of the section, relative to its center.

In order to construct the diagrams of the internal efforts, the corresponding "floor" of beam is divided into separate portions, within which the forces are characterized by continuous functions. The boundaries of these portions are the points of application of external force actions and support reactions, as well as cross-sections in which the application of distributed loads begins or ends.

In portions, where there is no distributed load, the bending moments change by to a linear law, and the shearing force is constant. Therefore, to construct the diagram of bending moments, it is enough to calculate the corresponding values in any two sections of the section, and to construct the diagram of shearing forces — in only one. On the diagram of bending moments the it's positive values are placed from below, that is, from the side of the tension fibers, and when constructing the diagram of shearing forces - from above. As a rule, the forces determined for the beams of individual "floors" are drawn on the general diagram of the entire hinged - cantilever beam.

As it is known from the course of strength of materials, there is a differential dependence between bending moments and shearing forces (Zhuravsky's theorem):

$$Q = \frac{dM}{dx}.$$

Based on the geometric content of the derivative, it can be said that the shearing force in the section is equal to the tangent of the angle between the axis of the beam and the tangent to the curve of bending moments.

In our consideration, we will go from the simplest case to a more complex one.

4.1. Determination of support reactions of a complex beam

A complex beam, which consists of two parts AC and DC, is shown on Fig. 1.29. These two parts are connected to each other by a spherical joint at a point C. The beam is fixed by a kinetic support (roller) at point A and it is fixed rigidly at another point D.

According Fig. 1.29, a uniformly distributed load of $q = 10$ kN is applied to the right part of the given beam, and a couple of forces with a moment of $M = 25$ kNm is applied at the point A.

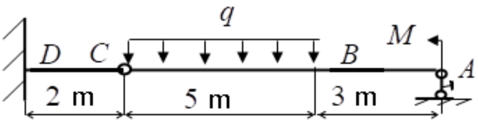


Fig. 1.29. The complex beam with loads.

Firstly we will run kinematic analysis for a given complex beam, and then determine the reactions at the attachment points A and D.

Finally, we will do a general verification of the obtained reactions.

1. Kinematic analysis.

1.1. **Quantitative stage.** The calculation scheme of the given mechanical system consists of three disks $Д_1$, $Д_2$ and $Д_3$. The disk $Д_3$ is the "earth" disk. The connecting elements of the scheme include soldering $П_{3.2}$, hinge $Ш_{2,1}$ and kinematic support C_1 (Fig. 1.30).

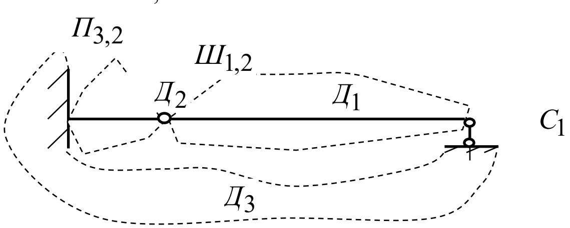


Fig. 1.30. The calculation scheme of the given complex beam.

Thus, we can write for a given construction that:

$$Д = 3, \qquad П = 1, \qquad Ш = 1, \qquad C = 1.$$

Then in this case, Chebyshov's formula will be written in the following form:

$$Г = 3 \cdot 3 - 3 \cdot 1 - 2 \cdot 1 - 1 - 3 = 9 - 9 = 0.$$

Therefore, the given calculation scheme is statically determined.

1.3. **Quality stage**. The given calculation scheme contains two stages of installation.

First stage (method of soldering). At this stage, the disks $Д_2$ and $Д_3$ are connected into one invariable disk D_1 using soldering $П_{3.2}$ (Fig. 1.31):

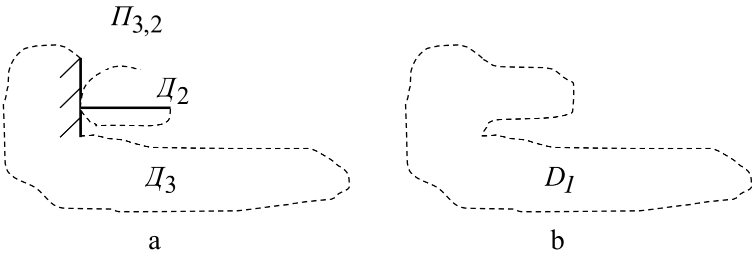


Fig. 1.31. The first stage of installation of given complex beam:
a – connection of two disks $Д_2$ and $Д_3$ by soldering $П_{3.2}$,
b – result of installation is invariable disk D_1.

$$\frac{Д_2 + Д_3}{П_{3,2}} \Rightarrow D_I.$$

Second stage (Polonso's method). At the second stage, the disk D_I is connected to the disk $Д_1$ using a hinge $Ш_{2,1}$ and a kinematic support C_1, As a result we obtain an invariable disk D_{II} (Fig. 1.32):

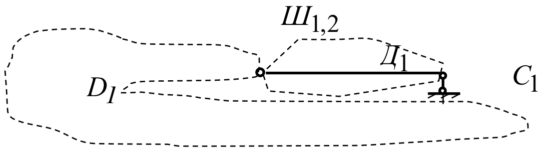


Fig. 1.32. The second stage of installation of given complex beam: connection of two disks D_I and $Д_1$ by hinge $Ш_{2,1}$ and kinematic support C_1.

In this case, we can write that:

$$\frac{Д_1 + D_I}{Ш_{1,2}, C_1} \Rightarrow D_{II}.$$

Conclusion: the given calculation scheme is statically determined and geometrically invariable system, which is constructed in two stages.

2. The given beam contains four unknown reactions: R_A, H_D, V_D and M_D (Fig. 1.33).

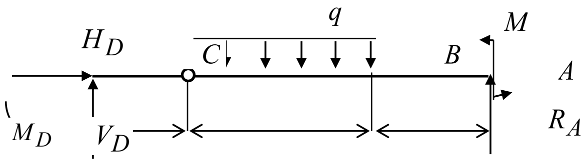


Fig. 1.33. The complex beam with support reactions.

If the given beam does not contain a spherical hinge, then it will consider as a statically indeterminate system. In this case, the equations of statics would not be sufficient to determine the reactions of the beam supports. The presence of a hinge allows cutting a given beam into two simple disks. From the equilibrium conditions of each of the disks, we will determine the above-mentioned reactions sequentially.

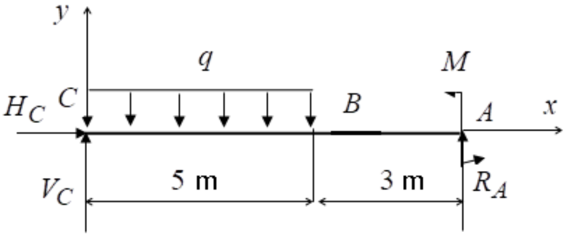


Fig. 1.34. The simple disk $Д_1$.

2.1. Let us consider the equilibrium of the disk $Д_1$. We make a calculation scheme, removing the supports from disk and applying to it the according reactions of them H_C, V_C and R_A. We denote the positive directions of the axes *x* and *y* (Fig. 1.34).

From the conditions of equilibrium, we determine the reactions of supports:

1. $\sum_{i=1}^{n} F_{ix}^{I} = 0$. $\quad H_C \equiv 0$.

2. $\sum M_A^I = 0$, $\quad -V_C \cdot CA + q \cdot CB\left(\dfrac{CB}{2} + BA\right) + M = 0$,

or

$$-V_C \cdot 8 + q \cdot 5 \cdot 5{,}5 + M = 0.$$

Where:

$$V_C = \frac{q \cdot 27{,}5 + M}{8} = \frac{10 \cdot 27{,}5 + 25}{8} = 37{,}5 \text{ kN};$$

3. $\sum M_C^I = 0$, $\quad R_A \cdot AC - q \cdot \dfrac{CB^2}{2} + M = 0$,

or

$$R_A \cdot 8 - q \cdot 5 \cdot 2{,}5 + M = 0.$$

Where:

$$R_A = \frac{q \cdot 12{,}5 - M}{8} = \frac{10 \cdot 12{,}5 - 25}{8} = 12{,}5 \text{ kN}.$$

Let us check the found reactions:

$$\sum_{i=1}^{n} F_{iy}^{I} = 0; \quad R_A + V_C - q \cdot 5 = 12{,}5 + 37{,}5 - 50 = 50 - 50 = 0.$$

2.2. Let us consider the equilibrium of the disk $Д_2$. We make a calculation scheme. We apply the determined reaction V_C as a known force in such a way that the equilibrium of the hinge C is saved. We denote the positive directions of the axes *x* and *y* (Fig. 1.35).

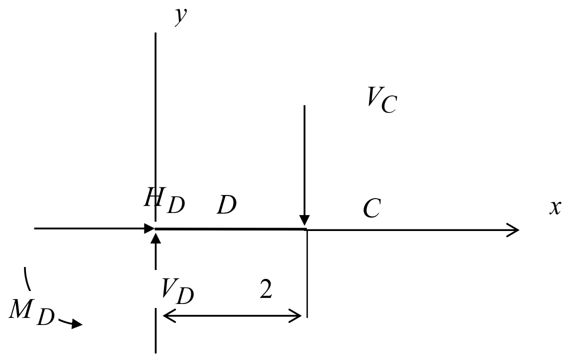


Fig. 1.35. The simple disk $Д_2$.

Then:

1. $\sum_{i=1}^{n} F_{ix}^{II} = 0.$ $\quad H_D \equiv 0.$

2. $\sum_{i=1}^{n} F_{iy}^{II} = 0$. $V_D - V_C = 0$.   Where:

$$V_D = V_C = 37{,}5 \text{ kN.}$$

3. $\sum M_D^{II} = 0$, $M_D - V_C \cdot CD = 0$,   or $M_D - V_C \cdot 2 = 0$

Where:

$$M_D = V_C \cdot 2 = 37{,}5 \cdot 2 = 75 \text{ kNm.}$$

3. We will run a general check of the found reactions:

1. $\sum_{i=1}^{n} F_{ix} = 0$. $H_D + H_C = 0 + 0 \equiv 0$;
2. $\sum_{i=1}^{n} F_{iy} = 0$. $V_D - q \cdot CB + R_A =$

$$= 37{,}5 - 10 \cdot 5 + 12{,}5 = 50 - 50 = 0;$$

3. $\sum M_C = 0$, $M_D - V_D \cdot 2 - q \cdot 5 \cdot 2{,}5 + M + R_A \cdot 8 =$

$$= 75 - 37{,}5 \cdot 2 - 10 \cdot 12{,}5 + 25 + 12{,}5 \cdot 8 =$$

$$= 75 - 75 - 125 + 25 + 100 = 200 - 200 = 0.$$

So, the reactions are defined correctly.

4.2. The construction of internal enforces diagrams for complex beams

We apply the determined reactions of the support to the beam and construct a diagram of shearing forces Q_x (Fig. 1.36, b):

I portion. $0 \le x_1 \le 3$ m. $Q_{x_1} = -R_A = -12{,}5$ kN.

II portion. $3 \le x_2 \le 8$ m. $Q_{x_2} = -R_A + q(x_2 - 3)$,

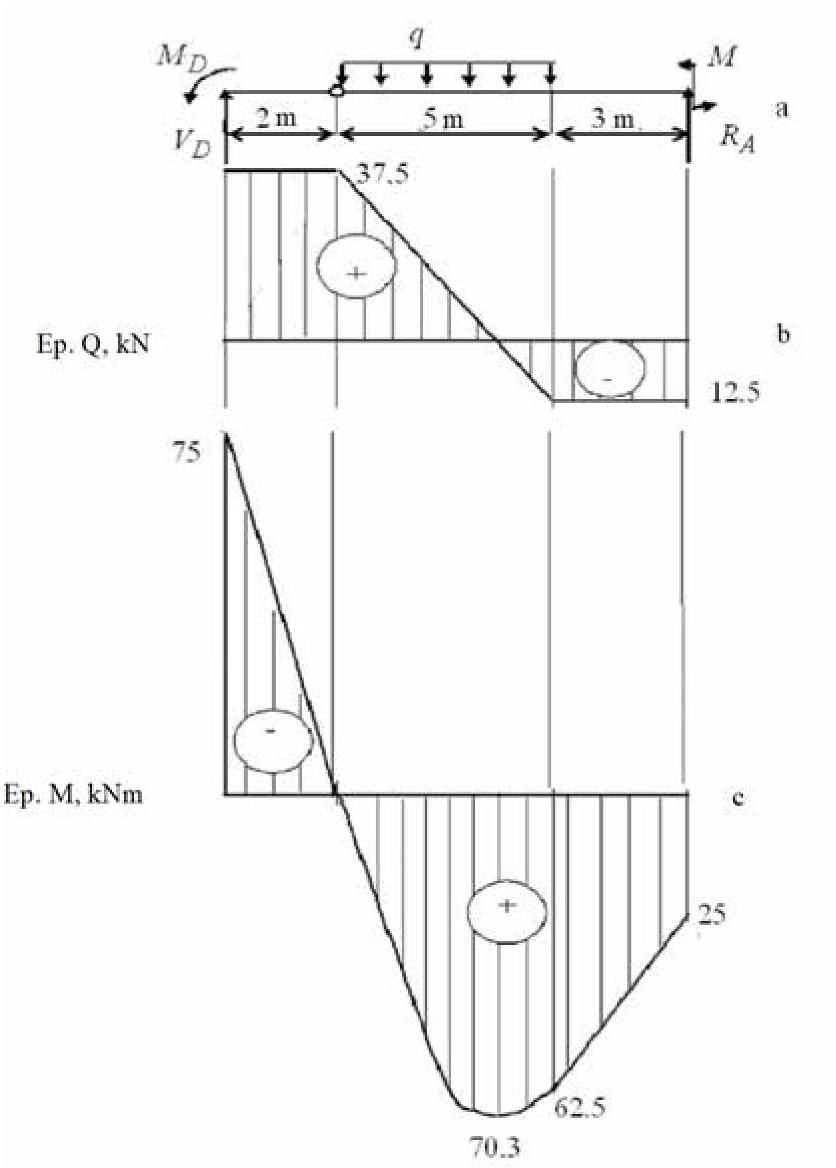


Fig. 1.36. The diagrams of internal efforts of complex beam:
a – complex beam, b – the diagram of shear force,
c – the diagram of bend moment

or:

$$Q_{x_2} = -12{,}5 + 10(x_2 - 3).$$

Then:

$$Q_{x_2}(3) = -12{,}5 \text{ kN}; \quad Q_{x_2}(8) = -12{,}5 + 50 = 37{,}5 \text{ kN}.$$

III portion. $0 \le x_3 \le 2$ m. $\qquad Q_{x_3} = V_D = 37{,}5$ kN.

Let us build the diagram of bending moments M_x (рис. 1.36, c):

I portion. $0 \le x_1 \le 3$ m. $\qquad M_{x_1} = M + R_A \cdot x_1 = 12{,}5 \cdot x_1$.

$M_{x_1}(0) = 25$ kNm; $\qquad M_{x_1}(3) = 62{,}5$ kNm.

II portion. $3 \le x_2 \le 8$ m. $\qquad M_{x_2} = M + R_A \cdot x_2 - q\dfrac{(x_2 - 3)^2}{2}$,

$M_{x_2}(5) = 62{,}5$ kNm; $\quad M_{x_2}(9) = 25 + 12{,}5 \cdot 8 - 5 \cdot 25 = 0$ kNm;

$$M_{x_2}(4{,}25) = 25 + 12{,}5 \cdot 4{,}25 - 5 \cdot 1{,}25^2 = 70{,}3 \text{ kNm}.$$

III portion. $0 \le x_3 \le 2$ m. $\qquad M_{x_3} = -M_D + V_C \cdot x_3$.

$M_{x_3}(0) = -75$ kNm; $\quad M_{x_3}(2) = -75 + 37{,}5 \cdot 2 = 0$ kNm.

Self-control questions

1. *What are the main stages of kinematic analysis of a complex beam do you know?*

2. *What the simple disk has a place in the calculation scheme of a complex beam always?*

3. What is the principle of determining the reactions of supports for the complex beams?

4. Please, formulate the principles of construction of the diagram of shearing forces for complex beams.

5. What are the features of constructing a diagram of bending moments for complex beams?

THEME 5.
CONSTRUCTION OF LINES OF INFLUENCE OF SUPPORT REACTIONS FOR STATICLY INDETERMINED BEAMS

5.1. The construction of lines of influence of support reactions of a simple support beam

5.2. The construction of lines of influence of support reactions of a cantilever beam

Sometimes there is a need to assess the stress-strain state of the structure from the so-called moving load. A moving load means that a load that continuously changes its position on the structure. For example, we can meet a moving load on the bridges (Fig. 1.37). Since various types of transport can be attributed to such a loads. Therefore, we have to calculated these constructions by moving load.

Fig. 1.37. The moving load on the bridges

The external load, which changes its position on the structure, causes movement and forces in it: reactions of supports, internal forces, stresses. This load can be classified as a dynamic load.

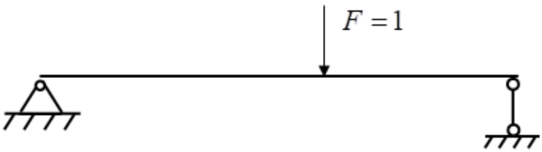


Fig. 1.38. A unite load moves along the simple supported beam

Thus, the forces in one or another element of the structure depend on the position of the moving load. To determine the calculated values of forces, it is necessary to choose from all possible load positions the one in which the calculated element will be in the most unfavorable conditions. So, when choosing the dimensions of the cross-section of the beam, the given moving load must be placed in such a way as to obtain the maximum force in the given cross-section. This position of the load is called dangerous. Therefore, each cross-section of the beam corresponds to its dangerous position of the moving load.

We will study the action of the moving load on the structural elements by considering the simplest case, when only a unite load moves along the simple supported beam (concentrated force (Fig. 1.38)).

5.1. The construction of lines of influence of support reactions of a simple support beam

First, let us consider the problem about the support reactions change by acting of moving load. We will plot the law of the change of reaction depending on the position of the moving load $F=1$ graphically.

The plot, which reflects the law of change of a certain factor when moving along the structure of the force $F=1$ is called **the line of influence** of this factor.

Comment. It is necessary to distinguish the line of influence from the diagram of internal efforts. These are inherently opposite concepts. The ordinates of the diagram characterize the distribution of this factor over different cross-sections of the beam under a static load. The ordinates of the line of influence, on the contrary, characterize the change in the factor that occurs in a specific cross-section when the force $F=1$ moves along the beam.

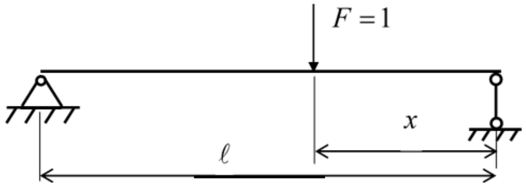


Fig. 1.39. The distance from the right support to the load is x.

Let the load $F=1$ is moving along the simple support beam (Fig. 1.39).

We denote the distance from the right support to the load by x. This distance will change from zero, when the load is on the right support, to parameter ℓ, when the load will locate on the left support.

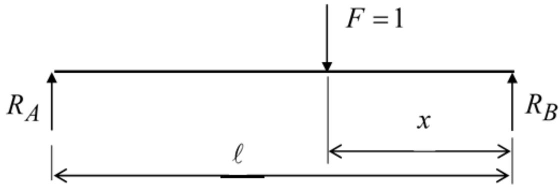


Fig. 1.40. The simple beam with supports.

Let us determine the value of the reactions R_A of the left support depending on the distance (Fig. 1.40). To do this, we write down the equilibrium equation, namely: the sum of the moments of all forces relative to the right support, we get:

$$\Sigma M_B = -R_A \cdot \ell + F \cdot x = 0,$$

or

$$R_A = \frac{F \cdot x}{\ell},$$

given that $F=1$, we finally get:

$$R_A = \frac{x}{\ell}. \qquad (1.8)$$

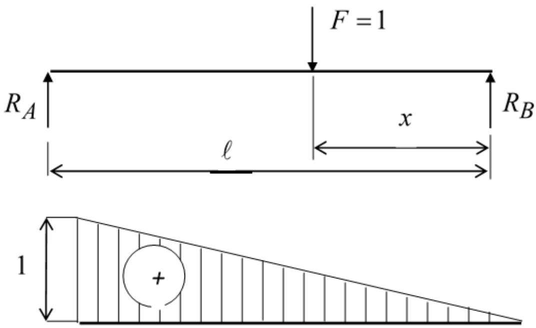


Fig. 1.41. The line of influence of the support reaction R_A.

Equation (1.8) is the law of the change in the magnitude of the reaction R_A depending on the position of the load $F = 1$. Depicting it graphically, we get the line of influence of the support reaction R_A. Since the variable x in equation (1.8) is included in the first power, the line of influence will be straight (Fig. 1.41). We build it according to two points:

at $x = 0$, we get $R_A = 0$; at $x = \ell$ we get $R_A = 1$.

If a load acts on the beam, then to calculate the support reaction from this load, the ordinate of the influence line measured under the load must be multiplied by the value F_1. In the case of simultaneous action on the beam of several concentrated vertical forces (loads), it is necessary to find the numerical values F_1 of the support reactions separately from each force, and then by adding the reactions R_A from individual forces, get the full reaction value from the given system of concentrated forces.

Let us get the law of change of support reaction R_B. To do this, we write down the sum of the moments of all forces relative to the left hinge, we get:

$$\sum M_A = R_B \cdot \ell + F \cdot (\ell - x) = 0,$$

where

$$R_B = \frac{F \cdot (\ell - x)}{\ell},$$

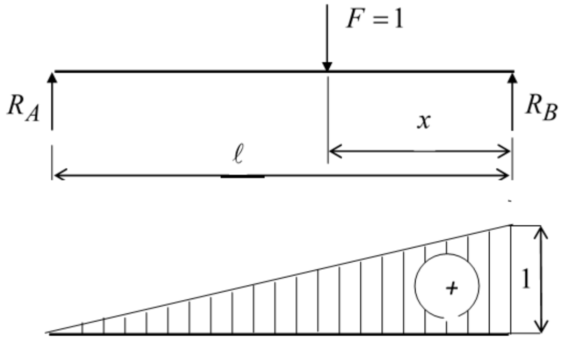


Fig. 1.42. The line of influence of the support reaction R_B.

if we will taking into account that $F = 1$, we finally get:

$$R_B = \frac{\ell - x}{\ell}. \tag{1.9}$$

Equation (1.9) is the law of changing the numerical value of the reaction R_B, when the load $F = 1$ is moving. Let us construct the line of influence of the support reaction R_B, it will be in the form of a straight line as in the previous case (Fig. 1.42). We build it according to two points:

at $x = 0$, we get $R_B = 1$;   at $x = \ell$ we get $R_B = 0$.

If we analyze the influence lines R_A, which is shown in Fig. 1.39, then it is possible, for example, to draw such a conclusion that in order for the reaction to have the greatest value F_1, it is enough to place the load above the left support (above the largest ordinate of the line of influence R_A).

Consider the construction of the lines of influence of reactions R_A and R_B for a beam on two supports with cantilevers, shown in Fig. 1.43.

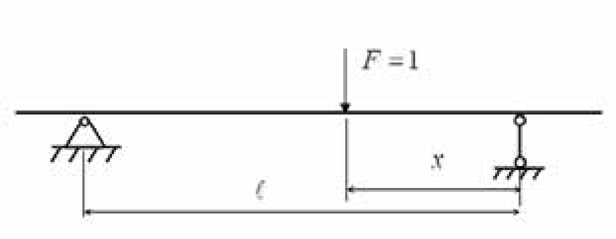


Fig. 1.43. The beam on two supports with cantilevers.

It is easy to see that the reaction equations will be the same as for the beam shown in Fig. 1.39. If we will continue the straight lines that limit the lines of influence on the cantilevers, we get the lines of influence R_A and R_B, which are shown in Fig. 1.44, b, c. Negative ordinates of influence lines $F=1$ of supporting reactions mean that when the load is located above them, the support reactions R_A and R_B are negative, that is, directed downwards.

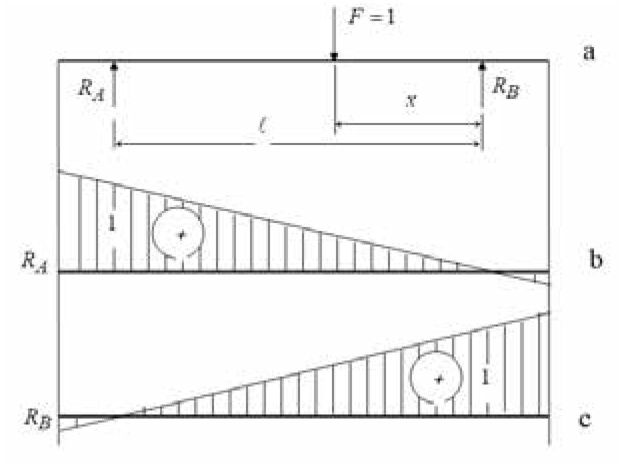


Fig. 1.44. The construction of lines of influence of support reactions for beam on two supports with cantilevers:
a – given beam, b - lines of influence of support reaction R_A,
c - lines of influence of support reaction R_B.

5.2. The construction of lines of influence of support reactions of a cantilever beam

Next, we will construct the influence lines of the support reactions for the cantilever beam, which is shown in Fig. 1.45. Two reactions arise from a vertical unit force in a rigidly fixed beam: vertical reaction R_A and reaction moment M_A (Fig. 1.46).

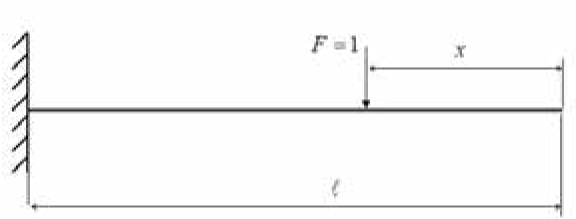


Fig. 1.45. The cantilever beam with moving loads.

First, let us build a line of influence R_A. From the equilibrium equation of the projection of all forces on the vertical axis, we obtain:

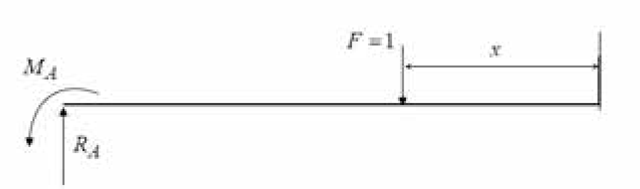


Fig. 1.46. The reactions of cantilever beam with moving loads.

$$\Sigma F_y = R_A - 1 = 0,$$

where

$$R_A = 1. \qquad (1.16)$$

It means that with an arbitrary position of the load $F = 1$, the reaction R_A is equal to unity. The corresponding line of influence is presented in Fig. 1.47, b.

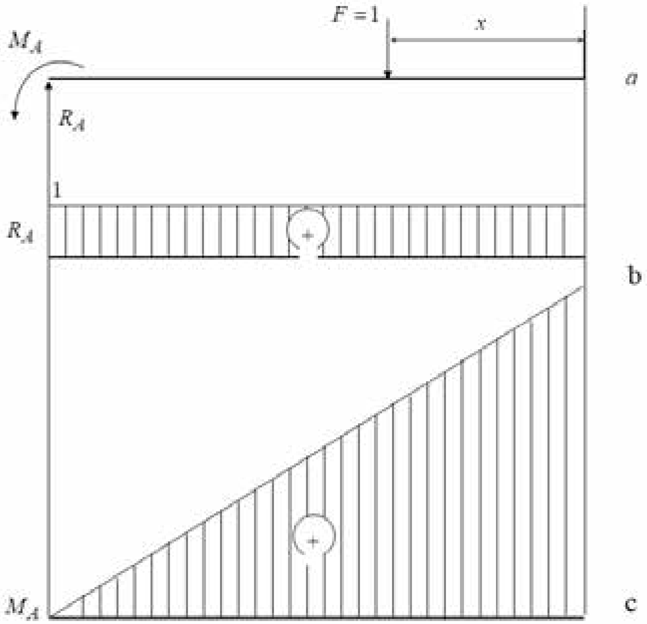


Fig. 1.47. The construction of lines of influence of support reactions for cantilever beam: a – given beam,
b - lines of influence of support reaction R_A,
c - lines of influence of support reaction M_A.

Next, consider the construction of the line of influence M_A. From the equilibrium condition $\sum M_A = 0$, we get:

$$M_A - 1 \cdot x = 0,$$

where:

$$M_A = x. \tag{1.17}$$

Then:

$$M_A(0) = 0; \qquad M_A(\ell) = \ell.$$

Thus, the ordinates of the lines of influence of the bending moment has the dimension of length. Therefore, the scale for the ordinates of the lines of influence of the reaction moment can be chosen the same as for the length of the beam. The line of influence M_A is present in fig. 1.47, c.

Self-control questions

1. *How do we can take into account the impact of a moving load on calculation of structure in construction mechanics?*

2. Is the speed of the moving load taking into account, when we are constructing the influence lines?

3. Reveal the physical meaning of the concept of the line of influence.

4. What is the law describe the lines of influence for the reactions R_A *of a simple support beam?*

5. What is the law describe the lines of influence for the reactions R_B *of a simple support beam?*

6. What is the section of a simple support beam the line of influence of the shearing force has a leap?

7. What is the law describe the lines of influence for the reactions R_A *of a cantilever beam?*

8. What is the law describe the lines of influence for the reactions M_A *of a cantilever beam?*

THEME 6.
THE CONSTRUCTION OF LINES OF INFLUENCE OF BENDING MOMENTS AND SHEARING FORCES FOR STATICLY INDETERMINED BEAMS

6.1. Construction of the influence line of the bending moment for a simple supported beam.

6.2. Construction of the line of influence of the shearing force for a simple supported beam.

6.3. Construction of the influence line of the bending moment for the cantilever beam.

6.4. Construction of the line of influence of the shearing force for a cantilever beam.

6.1. Construction of the influence line of the bending moment for a simple supported beam

Let us consider the ways of constructing of influence lines for a beam on two supports. We will begin the consideration by constructing the lines of influence of the bending moment for the cross section, which is at a distance from the left support and the distance from the right support (Fig. 1.48).

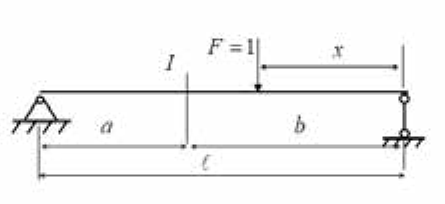


Fig. 1.48. The simple beam with the cross-section *I.*

As it is known, from the course of strength of materials, the bending moment, which is acting in the cross-section of the beam, is equal to the algebraic sum of the moments of the external left forces relative to the centroid of the given section. It may be the sum of the moments of the right forces, which are taken with the opposite sign.

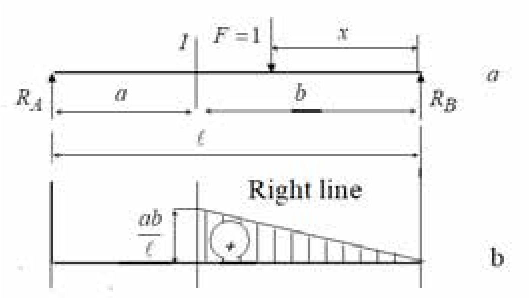


Fig. 1.49. The construction of right line of line of influence M_I:
a - the load is located to the right of the cross-section *I,*
b - right line of line of influence M_I.

In the case, when the load is located to the right of the cross-section *I*, that is, while the condition is satisfied that $x \le b$, of the external forces to the left of the cross-section *I*, only the reaction R_A is applied to the beam (Fig. 1.49 a), Then the moment M_I in the cross-section *I* is equal to:

$$M_I = R_A \cdot a. \qquad (1.18)$$

Accordingly, the line of influence M_I can be obtained from the line of influence R_A by multiplying it's ordinates on the distance a. Substituting the reaction value R_A (expression (1.8)) in equation (1.18), we get:

$$M_I = \frac{xa}{\ell}. \qquad (1.19)$$

To construction of the plot of equation (1.19), we calculate the value of the moment M_I at two points:

$$M_I(0) = 0; \qquad M_I(b) = \frac{ab}{\ell}. \qquad (1.20)$$

Based on the expressions (1.20), we construct a straight line, which is called the right line of the influence line M_I (Fig. 1.49, b). Its ordinates give the value of the bending moment in the section *I*, when the load is located to the right of this section, i.e. at $x \le b$.

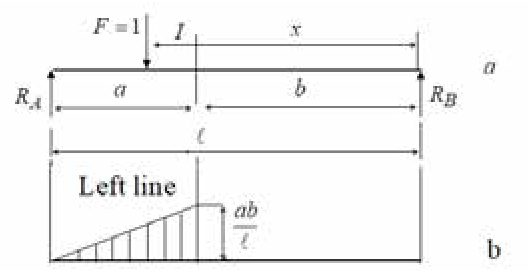


Fig. 1.50. The construction of left line of line of influence M_I:
a - the load is located to the left of the cross-section *I,*
b – left line of line of influence M_I.

When the load is located to the left of the section *I* (Fig. 1.50, a), i.e. at $x \geq b$, to determine the bending moment in the beam section, it is more convenient to consider its right side. Then:

$$M_I = R_B \cdot b. \qquad (1.21)$$

Substituting the value of reaction R_B into expression (1.21), we obtain:

$$M_I = \frac{\ell - x}{\ell} b.$$

(1.22)

To construct the plot of expression (1.22), we will again calculate two values of the moment M_I, but at another points, namely:

$$M_I(b) = \frac{\ell - b}{\ell} b; \;\; M_I(\ell) = \frac{\ell - \ell}{\ell} b = 0. \qquad (1.23)$$

Based on the expressions (1.23), we construct a straight line, which is called the left straight line of the influence line M_I (Fig. 1.50, b). Its ordinates give the value of the bending moment in the section *I*, when the load $F = 1$ is located to the left of this section. That is, when $b \leq x \leq \ell$.

If the two parts of the line of influence (Fig. 1.49, b and Fig. 1.50, b) are connected, then both straight lines will cross under the section *I* and as a result, we get the line of influence M_I (Fig. 1.51).

Based on the above, we can build a line of M_I in the following way: at first we build one of the straight lines, for example the right one, and then, we build the left straight line, connect the zero point of the left support with the point of the right straight line located under the section *I*.

The ordinate of the influence line M_I will give the numerical value of the bending moment in the section *I*, when the load $F = 1$ is located above this ordinate. Accordingly, in order to obtain the numerical value of the bending moment M_I in the section *I* at a given position of the load $F = 1$, it is necessary to calculate the ordinate of the line of influence under the load. Note that the line of influence M_I gives the law of change of the bending moment in the section *I*. In order to obtain the law of change of the bending moment in another arbitrary cross-section, it is necessary to construct the line of influence for this cross-section.

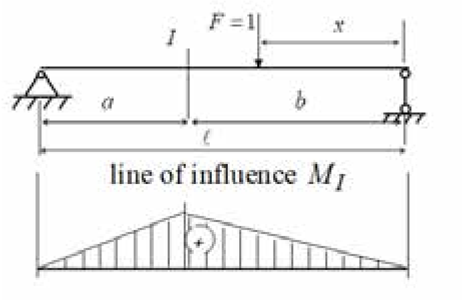


Fig. 1.51. The line of influence M_I .

6.2. Construction of the line of influence of the shearing force for a simple supported beam

Let us proceed to consider the construction of the line of influence of the shearing force, which occurs in the section *I*. As we know, from the course of strength of materials, the shearing force acting in a given cross-section is equal to the algebraic sum of the projections of external forces applied to the left of the given cross-section of the beam, on the normal to the axis of the beam. So, let us consider two positions of the load $F = 1$ again.

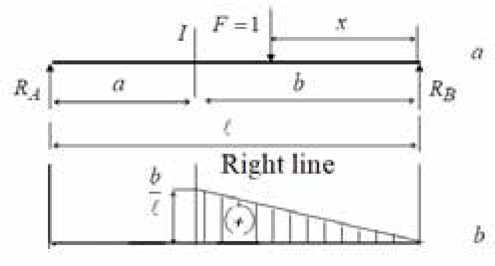


Fig. 1.52. The construction of right line of line of influence Q_I :
a - the load is located to the right of the cross-section *I,*
b - right line of influence line Q_I .

In the first case, the load is located to the right of the section *I*, i.e. $x \leq b$ (Fig. 1.52, a). From the balance of the left part of the beam, we get:

$$Q_I = R_A = \frac{x}{\ell}. \tag{1.24}$$

To construct the plot of equation (1.24), we calculate the values of shearing force at two points, namely:

$$Q_I(0) = 0; \qquad Q_I(b) = \frac{b}{\ell}. \tag{1.25}$$

Based on expressions (1.25), we construct the right straight line of influence Q_I, which is presented in Fig. 1.52, b.

In the second case, the load is located to the left of the section *I*, i.e. $x \geq b$ (Fig. 1.54, a). Considering the right part of the beam, we get:

$$Q_I = -R_B. \tag{1.26}$$

Taking into account the expression for (1.9), we obtain:

$$Q_I = -\frac{\ell - x}{\ell}. \tag{1.27}$$

Let us calculate two values of shearing force Q_I again:

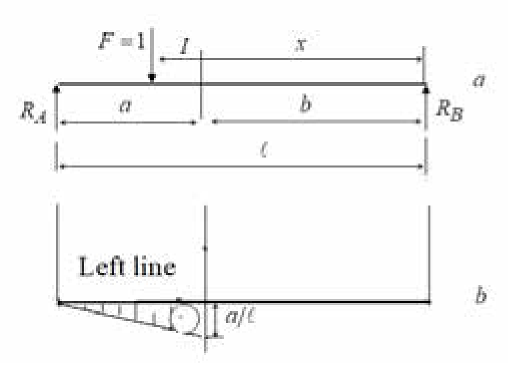


Fig. 1.54. The construction of left line of line of influence Q_I:
a - the load is located to the left of the cross-section *I*,
b - left line of influence line Q_I.

$$Q_I(b) = -\frac{a}{\ell}; \qquad Q_I(\ell) = 0. \qquad (1.28)$$

According to the expressions (1.28), the left straight line of influence Q_I was constructed, which is presented in Fig. 1.54, b.

The line of influence Q_I has the following feature: if we extend the straight lines that limit the line of influence Q_I to the supports, then we will get 1 under the left support of the ordinate, and -1 under the right support of the ordinate (Fig. 1.54, b). Therefore, there is an alternative way of building a line of influence Q_I. It can constructed such as follows: an ordinate equal to 1 is placed on the left reference vertical, and an ordinate equal to -1 is placed on the right reference vertical, while their vertices are connected to the zero points of the left and right supports. In this way, two parallel lines were obtained. Then it is necessary to demolish the section I as shown in Fig. 1.54, b.

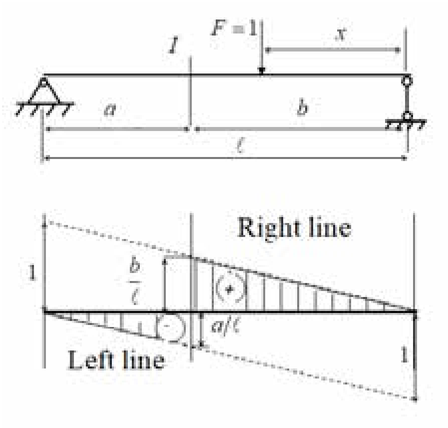


Fig. 1.55. The line of influence Q_I.

As can be seen from fig. 1.55, the line of influence Q_I in the section I has a leap – breaking of function. The ordinates of the flow line of the shearing force are given values. Therefore, the scale of these ordinates will be the same as for the reference reactions. The ordinate of the influence line Q_I gives the numerical value of the shearing force in the section *I*, when the load $F=1$ is located above this ordinate. Thus, in order to obtain the numerical value of the shearing force in the section *I* at a given position of the load $F=1$, it is enough to measure the ordinate of the line of influence Q_I under the load.

If the ordinate under the load $F=1$ is negative, it means that the shearing force in the section *I*, at the given position of the load, is negative. The ordinates of the influence line Q_I characterize the changing in shearing force only for the section *I*. In order to obtain the law of change of shearing force in another arbitrarily selected cross-section, it is necessary to construct the line of influence anew for the selected cross-section.

6.3. Construction of the influence line of the bending moment for the cantilever beam

Consider the construction of lines of influence for the bending moment and shearing force for the cantilever beam shown in Fig. 1.56, a. Let us start with the construction of the line of influence M_I. Let us consider two cases of position of load $F=1$.

Let the load of $F=1$ will be state to the left of the section *I* (Fig. 1.56, a). Then:

$$M_I = 0. \tag{1.29}$$

When the load $F=1$ is located to the right of the section *I* (Fig. 1.53. b), then the law of change of bending moment M_I will look like this:

$$M_I = -F \cdot x, \tag{1.30}$$

where *x* is the distance from the load $F=1$ to the section *I*.

To construct of the plot of equation (1.30), we calculate the values at two points:

$$M_I(0) = 0; \qquad M_I(b) = -b.$$

The corresponding influence line of M_I is shown in Fig. 1.56, c.

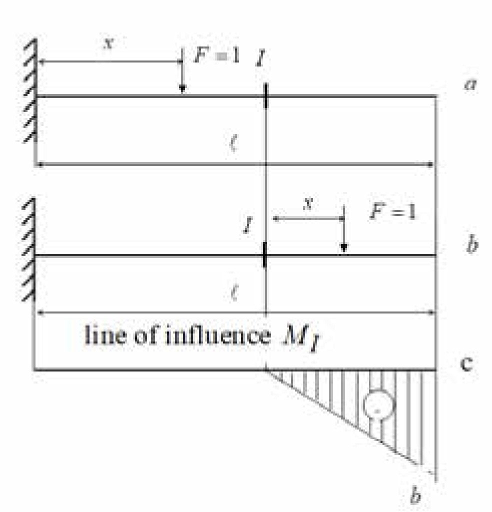


Fig. 1.56. The construction of of line of influence M_I:
a - the load is located to the left of the cross-section *I,*
b - the load is located to the right of the cross-section *I,*
c – the diagram of line of influence M_I.

6.4. Construction of the line of influence of the shearing force for a cantilever beam

Similarly to the previous case, to construct the line of influence of the shearing force Q_I in the cross-section *I*. Let us consider two positions of the load: when the load states to the left of the cross-section *I* and when the load states to the right of the cross-section *I*.

Position 1: the load is to the left of the section *I* (Fig. 1.57, a). Then the law of change of shearing force Q_I will be described by an equation of the form:

$$Q_I = 0. \qquad (1.31)$$

Since there are no forces on the left, then we get $Q_I = 0$.

Position 2: the load is to the right of the section *I* (Fig. 1.57, b). In this case, the law of change of shearing force Q_I will be described by an equation of the form:

$$Q_I \equiv 1. \tag{1.32}$$

that is, at a distance from the ordinate $0 \le x \le b$, the lines of influence remain constant and equal to 1.

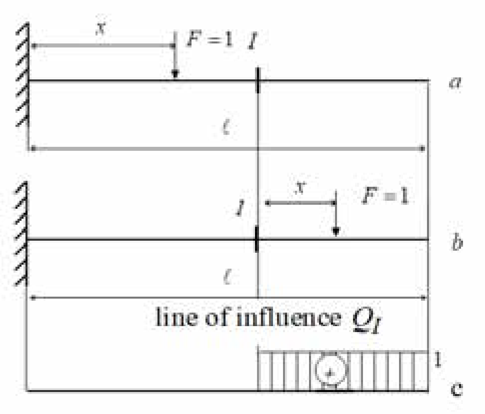


Fig. 1.57. The construction of of line of influence Q_I :
a - the load is located to the left of the cross-section *I,*
b - the load is located to the right of the cross-section *I,*
c – the diagram of line of influence Q_I .

The line of influence Q_I is shown in Fig. 1.57, c.

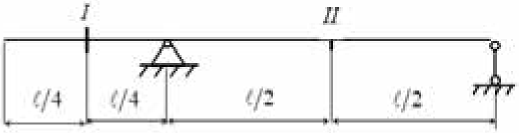


Fig. 1.58. The complex beam with cross-sections *I* and *II.*

To generalize our investigation that was presented in topic 1.6, we consider an example of constructing the lines of influence of internal forces for the beam, which is shown in Fig. 1.58.

At first, we will consider the construction of the lines of influence of the bending moment for a beam in cross-sections *I* and *II* for a simple support beam with cantilever, which is shown in Fig. 1.58.

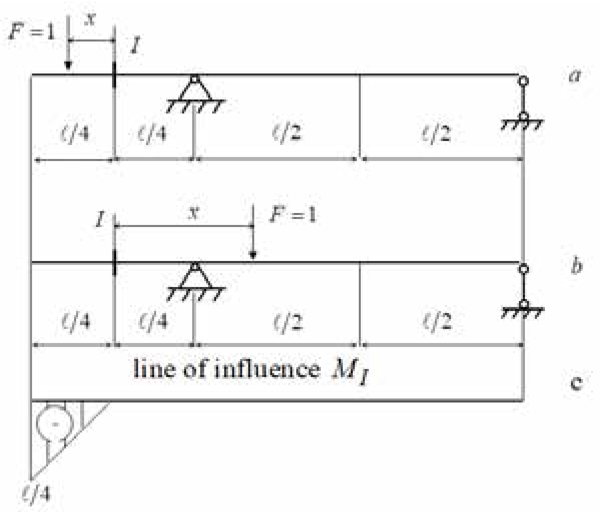


Fig. 1.59. The construction of of line of influence M_I:
a - the load is located to the left of the cross-section *I*,
b - the load is located to the right of the cross-section *I*,
c – the diagram of line of influence M_I.

We will begin the consideration with the construction of the lines of influence of the bending moment M_I. To construct the line of influence, it is necessary to consider two positions of the load $F = 1$: when the load is to the left of the section *I* and when the load is to the right of the section *I*.

Position 1: the load is to the left of the section *I* (Fig. 1.59, a). The law of changing of bending moment M_I will be:

$$M_I = -F \cdot x = -x .$$

To construct a line of influence in this region, we will consider the moment M_I at two points:

$$M_I(0) = 0; \qquad M_I\left(\frac{\ell}{4}\right) = -\frac{\ell}{4} .$$

Position 2: the load is located to the right of the section I (Fig. 1.59, b). The law of changing of bending moment M_I will be:

$$M_I \equiv 0.$$

The line of influence of the moment M_I is shown on Fig. 1.59, c.

At first, we will build the line of influence M_{II} as for a beam on two supports. For this, we consider three positions of the load $F = 1$. Position 1: this is the case, when the load locates to the left of the section *II* to the left support (Fig. 1.60, a). Position 2: this is the case, when the load locates to the left of the section *II* behind the left support (Fig. 1.60, b). Position 3: this is the case, when the load locates to the right of the section *II* (Fig. 1.60, c).

Position 1: the load locates to the left of the section *II* to the left support (Fig. 1.60, a). The law of changing of moment M_{II} will be:

$$M_{II} = \frac{x\ell/2}{\ell} = \frac{x}{2}.$$

For construction of a line of influence M_{II} in this region, we consider two points:

$$M_{II}(0) = 0; \qquad M_{II}\left(\frac{\ell}{2}\right) = \frac{\ell}{4}.$$

Position 2: the load locates to the left of the section *II* behind the left support (Fig. 1.60, b). The law of changing of moment M_{II} will be:

$$M_{II} = -F \cdot \frac{x}{\ell} \cdot \frac{\ell}{2} = -\frac{x}{2}.$$

We have to consider two points of construction of a line of influence M_{II} in this region:

$$M_{II}(0) = 0; \qquad M_{II}\left(\frac{\ell}{2}\right) = -\frac{\ell}{2}.$$

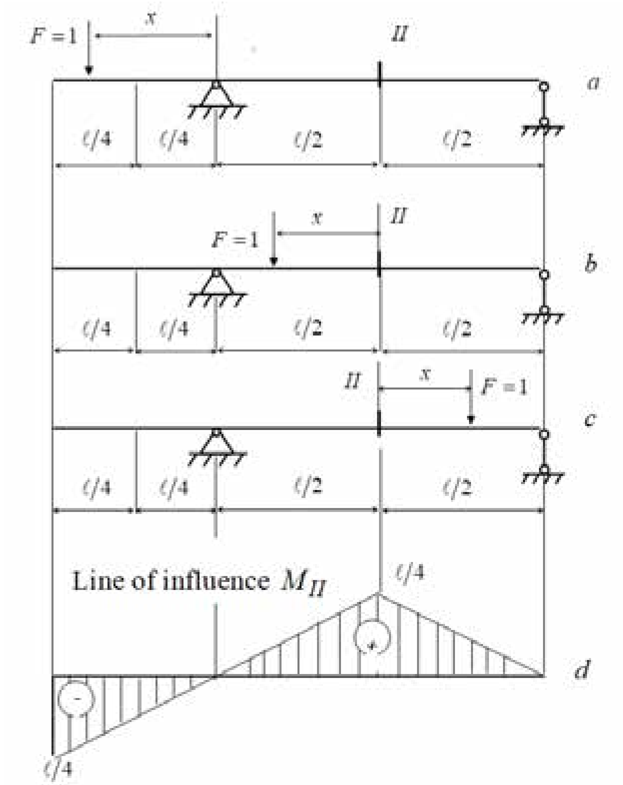


Fig. 1.60. The construction of of line of influence M_{II}:
a – the load is located to the left of left support,
b - the load is located to the left of the cross-section *II,*
c - the load is located to the right of the cross-section *II,*
d – the diagram of line of influence M_{II}.

Position 3: the load locates to the right of the section *II* (Fig. 1.60, c). The law of changing of moment M_{II} will be:

$$M_{II} = \frac{x\,\ell/2}{\ell} = \frac{x}{2}.$$

To construct a line of influence of bending moment M_{II}, we calculate its value at two points:

$$M_{II}(0) = 0; \qquad M_{II}\left(\frac{\ell}{2}\right) = \frac{\ell}{4}.$$

The line of influence of bending moment M_{II} is shown on Fig. 1.60, d.

For the beam, which is shown in Fig. 1.61 to construct the lines of influence of shearing forces Q_I and Q_{II}.

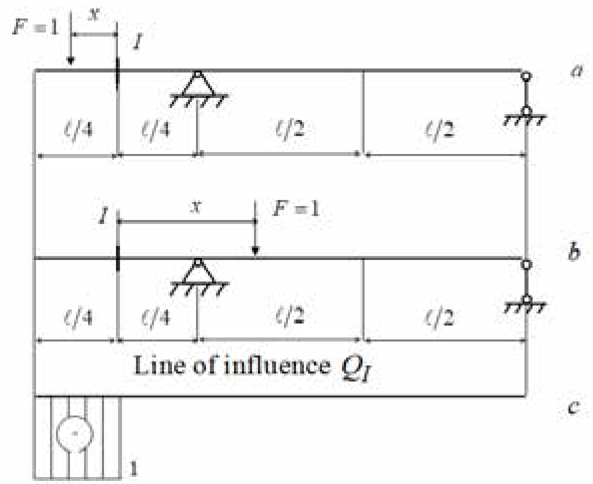


Fig. 1.61. The construction of of line of influence Q_I:
a - the load is located to the left of the cross-section *I,*
b - the load is located to the right of the cross-section *I,*
c – the diagram of line of influence Q_I.

We will begin our consideration with the construction of influence lines Q_I. To construct the line of influence Q_I, it is necessary to consider two positions of the load $F = 1$. The first position is the case, when the load is to the left of the section *I* and the second position is the case, when the load is to the right of the section *I*.

Position 1: the load locates to the left of the section *I* (Fig. 1.61, a). The law of changing of shearing force Q_I will be:

$$Q_I = -F = -1.$$

Position 2: the load locates to the right of the section *I* (Fig. 1.61, b). In this case the law of changing of shearing force Q_I will be:

$$Q_I \equiv 0.$$

The line of influence of shearing force Q_I is shown in Fig. 1.61, c.

At first, we will build the line of influence Q_{II} as for a beam on two supports, that is, we consider three positions of the load $F = 1$. Position 1 is the case, when the load locates to the right of the section *II* and moves to the right support. Position 2 is the case, when the load locates to the left of the section *II* in front of the left support. Position 3 is the case, when the load locates to the left of the section *II* behind the left support.

Position 1: the load locates to the right of the section *II* and moves to the right support (Fig. 1.62, a). The law of changing of shearing force Q_{II} will be:

$$Q_{II} = \frac{x}{\ell}.$$

To construct a line of influence in this area, consider two points:

$$Q_{II}(0) = 0; \qquad Q_{II}\left(\frac{\ell}{2}\right) = \frac{1}{2}.$$

Position 2: the load locates to the left of the section *II* in front of the left support (Fig. 1.62, b). The law of changing of shearing force Q_{II} will be:

$$Q_{II} = -\frac{x}{\ell}.$$

To construct a line of influence of shearing force Q_{II} in this region, we will consider two points:

$$Q_{II}(0) = 0; \qquad Q_{II}\left(\frac{\ell}{2}\right) = -\frac{1}{2}.$$

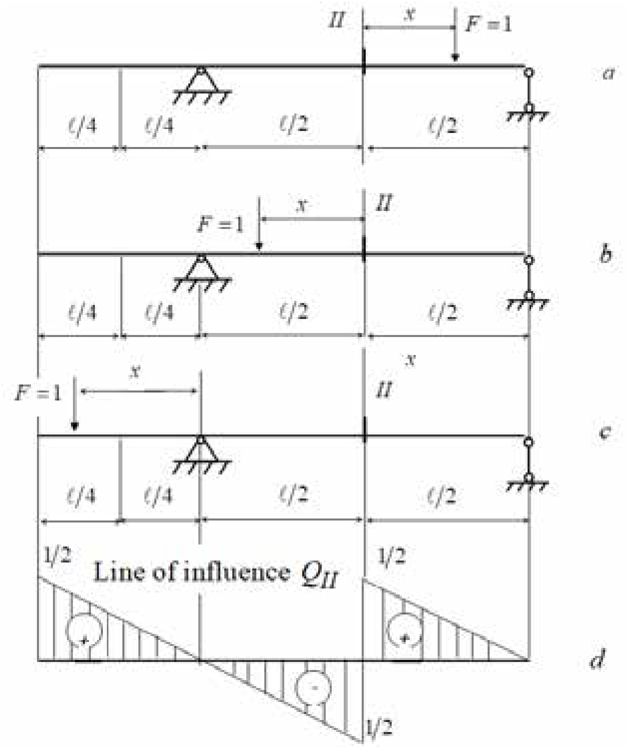


Fig. 1.62. The construction of of line of influence Q_{II} :
a – the load is located to the left of left support,
b - the load is located to the left of the cross-section *II,*
c - the load is located to the right of the cross-section *II,*
d – the diagram of line of influence Q_{II} .

Position 3: the load locates to the left of the section *II* behind the left support (Fig. 1.62, c). The law of changing of shearing force Q_{II} will be:

$$Q_{II} = \frac{\ell/2 - x}{\ell}.$$

To construct a line of influence of shearing force Q_{II} in this region, we will consider two points:

$$Q_{II}(0) = \frac{1}{2}; \qquad Q_{II}\left(\frac{\ell}{2}\right) = 0.$$

The line of influence of shearing force Q_{II} is shown in fig. 1.62, d.

Self-control questions

1. *Does the ordinates of the lines of influence of the bending moment have the dimension of length?*
2. *Has the diagram of influence line of shear force step for simple beam?*
3. *What is the law describe the lines of influence for shearing forces Q_I of a simple support beam?*
4. *What is the law describe the lines of influence for bend moment M_I of a simple support beam?*

THEME 7.
THE CONSTRUCTION OF INFLUENCE LINES OF INTERNAL EFFORTS FOR COMPLEX BEAMS

7.1. The construction of influence lines of internal efforts for complex by general method.
7.2. The construction of influence lines of internal efforts for complex beams by the kinematic method.

In structural mechanics, there is a statement about the straight linearity of the lines of influence between the joits of the mechanical system (construction). This statement can be extended to the case of complex statically determined beams.

Based on the above, we will consider the construction of lines of influence of internal efforts for complex beams using the examples of specific beams.

7.1. The construction of influence lines of internal efforts for complex by general method

Let us consider the construction of the lines of influence of the reactions for the beam, which is shown in Fig. 1.63. As can be seen from the figure, the given beam consists of two parts: simple beam CD, which is fixed at one end by kinematic support C and at the other one is hinged D with a simple support beam AB. The beam AB includes a cantilever.

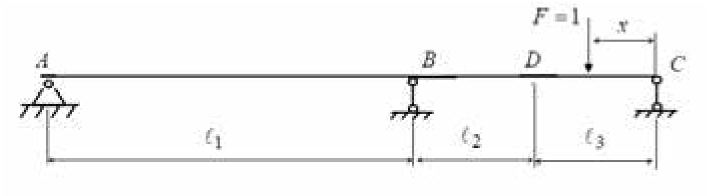


Fig. 1.63. Two-span beam.

It is advisable to consider two specific positions of the load $F=1$ by the construction of the line of influence R_C.

Position 1: it is the case, when the load locates on the beam CD. Its acting is distributed between points C and D in the same way as for a simple supported beam with length ℓ_3. The law of changing of reaction R_C is determined by the formula:

$$R_C = \frac{F \cdot (\ell_3 - x)}{\ell_3}. \qquad (1.33)$$

To construct the line of influence R_C, the expression (1.33) is calculated at two points:

$$R_C(0) = 1; \quad R_C(\ell_3) = 0.$$

Position 2: it is the case, when the load locates on beam with cantilever. In this case, the acting of load is distributed on points C and D is not transmitted. In this case the law of changing of reaction R_C will have the form:

$$R_C \equiv 0. \qquad (1.34)$$

The line of influence of reaction R_C is shown in Fig. 1.64.

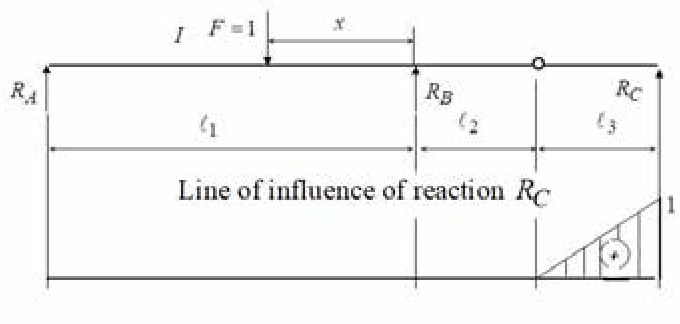


Fig. 1.64. The diagram of influence line of reaction R_C for beam by general method.

Let us construct the line of influence R_A. To this we will consider two positions of the load $F = 1$.

Position 1: the load moves along the beam AD. Then law of changing of reaction R_A has the form:

$$R_A = \frac{\ell_1 - x}{\ell_1}. \quad (1.35)$$

Then:

$$R_A(0) = 1; \qquad R_A(\ell_1) = 0; \qquad R_A(\ell_1 + \ell_2) = -\frac{\ell_2}{\ell_1}.$$

(1.36)

As we can see from the last expression (1.36), when the load locates at point D ($x = \ell_1 + \ell_2$), the support reaction is directed downward and reaches its largest negative value.

Position 2: the load moves along the beam DC. Then the acting of reaction R_A will be equal to $1 \cdot \frac{x}{\ell_3}$. It like such as it would be transmitted to this joint during the nodal transfer of the load on the portion DC.

Therefore, the line of influence R_A on the segment DC is a so-called transmission line. The line of influence R_A is shown in Fig. 1.65, a. The line of influence R_B is shown in Fig. 1.65, b accordingly.

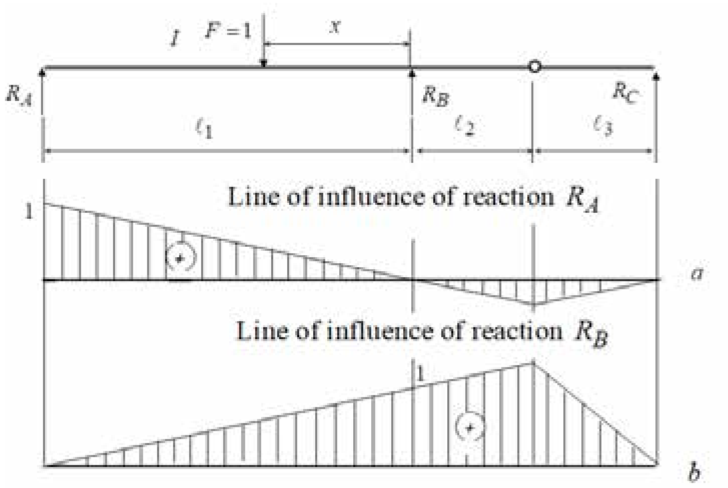


Fig. 1.65. The diagrams of influence line of reaction R_A and R_B by general method:
a - diagram of influence line of reaction R_A,
b - diagram of influence line of reaction R_B

7.2. The construction of influence lines of internal efforts for complex beams by the kinematic method

We will consider the construction of influence lines R_A and R_B for the beam shown in Fig. 1.66, a, by the kinematic method. This method base on the using of the principle of possible movements, which is studied in theoretical mechanics.

Let us remain the essence of the principle of possible movements: if the system states in equilibrium, then the sum of the work of all forces on arbitrary possible movements is equal zero.

Before proceeding to the construction of the influence line R_B, let us make some simplifications regarding the lengths of the beams AD and DC. Let we assume that $\ell_1 = 3a$, $\ell_2 = \ell_3 = a$. Then, in order to construct the line of influence of reaction R_B, we remove the kinematic support, which is applied at the point B, and denote the effort in this support by X (Fig. 1.66, b).

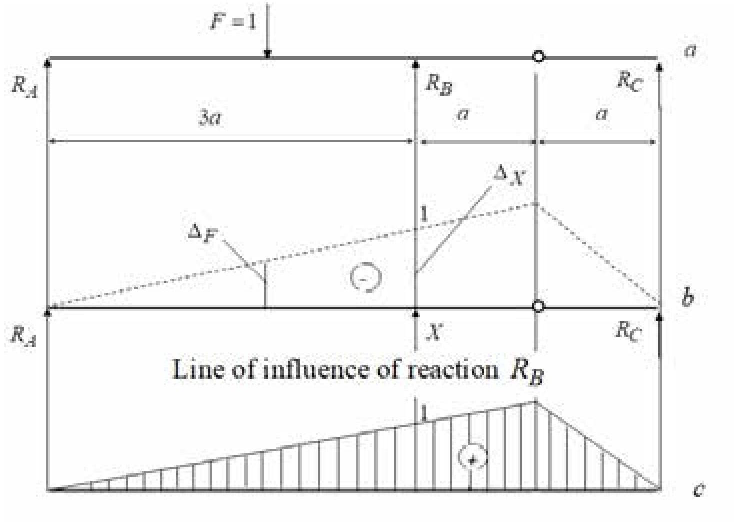


Fig. 1.66. The diagram of influence line of reaction R_B for complex beam by kinematic method:
a - complex beam with moving load,
b - possible displacement of given beam,
c - diagram of influence line of reaction R_B.

The given beam is a statically determined system. If we remove one support, it turned into a mechanical system with one degree of freedom. We give this system a possible displacement as shown in Fig. 1.66, b. At the same time, it is desirable to specify the possible displacement in such a way that the force run positive work, that is, the direction of displacement must coincide with the direction of the force.

According to the principle of possible displacements, we write down:

$$1 \cdot \Delta_F + X \cdot \Delta_X = 0, \qquad (1.37)$$

where:

$$X = -\frac{\Delta_F}{\Delta_X}, \qquad (1.38)$$

where Δ_F - displacement in the direction of the force $F = 1$, the numerical value of which depends on the position of the force.

Since the force $F=1$ moves through the entire beam, Δ_F is a set of displacements of all points of the beam vertically. It follows that that Δ_F possible displacement is a diagram. It is shown in Fig. 1.66, c. This plot has a sign of minus, since the displacement has the direction is opposite of the force $F=1$.

Dividing all the ordinates of this plot by parameter Δ_X and changing the sign to the opposite according to formula (1.38), we get the line of influence R_B, which is shown in Fig. 1.66, c. After changing the sign, the ordinates of the influence lines are placed in the same direction as the ordinates of the plot Δ_F, since the positive ordinates of the influence lines are placed upwards.

It should be noted that at the point of application of the force X, the ordinate of the line of influence is equal to unity, since in this case the equality is fulfilled:

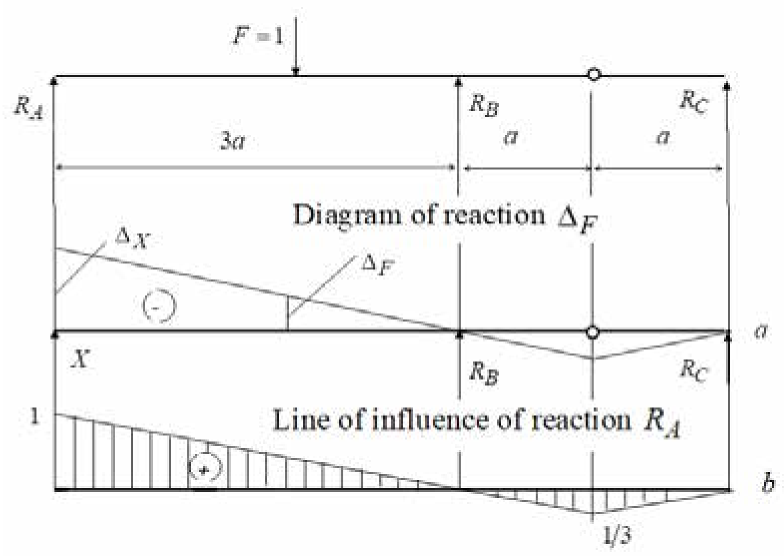


Fig. 1.67. The diagram of influence line of reaction R_A for complex beam by kinematic method:
a - possible displacement of given beam,
b - diagram of influence line of reaction R_A.

$$\Delta_F = -\Delta_X . \qquad (1.39)$$

In a similar way, the line of influence R_A was constructed, which is shown in Fig. 1.67, b.

Concluding consideration of this example, let us generalize about the kinematic method of constructing influence lines. Thus, in order to construct the line of influence of beam reactions using the kinematic method, the following actions must be run:

— remove the support, the line of influence of which must be built, replacing it with the action of force X ;

— to give the obtained system a possible displacement in such a way that the force X run a positive work, and to obtain an outline of possible displacements Δ_F ;

— by dividing all ordinates of the diagram Δ_X by Δ_X , obtain the line of influence of the desired effort.

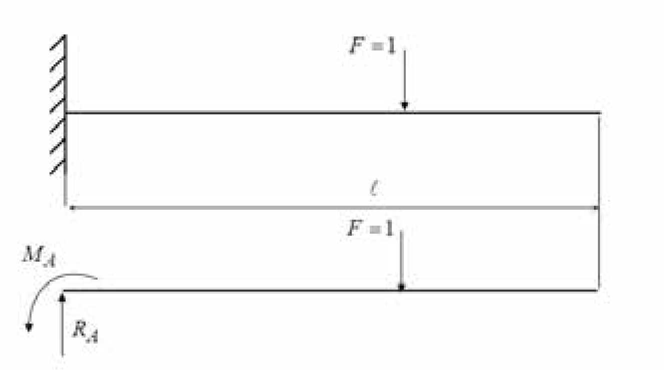


Fig. 1.68. The reactions of cantilever beam with moving loads.

For the completeness of the consideration, it will be appropriate to construct the influence lines of the support reactions for the beam, one end of which is fixed with solder, and the other is free (Fig. 1.68). Therefore, the support reactions of this beam are represented by reaction R_A and moment M_A (Fig. 1.68).

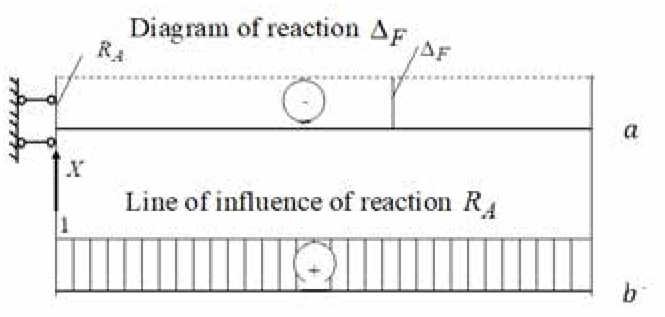


Fig. 1.69. The construction of diagram of influence line of reaction R_A for cantilever beam by kinematic method:

a - possible displacement of given beam,

b - diagram of influence line of reaction R_A.

At first, let us build a line of influence of reaction R_A. To do this, it is necessary to remove the corresponding support to the vertical displacement at the point A, while keeping all other supports. Therefore, unlike the previous beam, it is necessary to apply two horizontal kinematic supports as shown in Fig. 1.69, a. These kinetic supports made absent of rotation and horizontal displacement of the point A.

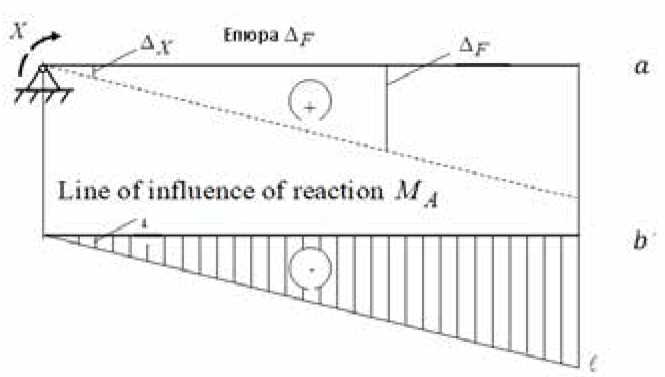


Fig. 1.70. The construction of diagram of influence line of reaction M_A for cantilever beam by kinematic method:

a - possible displacement of given beam,

b - diagram of influence line of reaction M_A.

Let us apply a vertical force X at the point A. We give the beam a possible displacement and build a diagram Δ_F (Fig. 1.69, a). Since equality (1.39) is fulfilled, if we multiply the diagram Δ_F by minus, we will gives us the line of influence of reaction R_A (Fig. 1.69, b).

To construct the line of influence of the moment M_A at the point A, we replace the rigid fastening with a hinge and apply the moment X (Fig. 1.70, a). The difference of this problem is the fact that the role of displacement Δ_X is the angle of rotation. Similarly, to the previous case, we build the line of influence M_A shown in Fig. 1.70, b.

Let us finish our consideration of the construction of influence lines for complex beams by considering the process of construction of the influence lines of internal forces using the kinematic method.

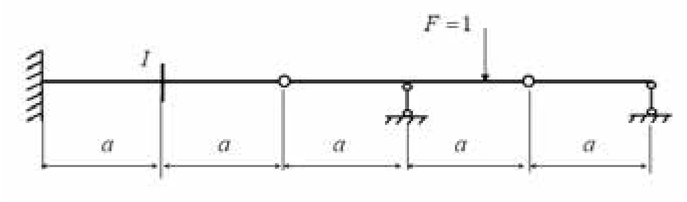


Fig. 1.71 The complex beam.

To construct the line of influence of the shearing force, we use the diagram shown in Fig. 1.72. After removing the support that corresponds to the shearing force, we apply the desired force X in such a way that its direction coincides with the positive direction of the shearing force.

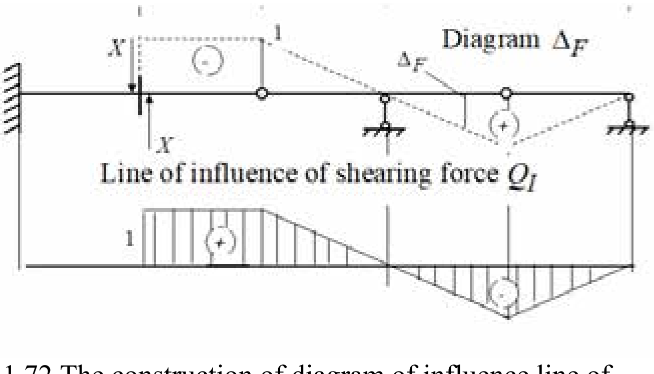


Fig. 1.72 The construction of diagram of influence line of shear force Q_I by kinematic method.

The difference in the construction of the lines of influence of internal forces from the lines of influence of supporting reactions lies in the fact that it is necessary to apply not one force, but two. One of the forces corresponds to the action of the right part of the beam on the left, and the other - vice versa. In this case, the displacement in the direction of the left force is zero. Therefore, we set the unit possible displacements in the direction of the force X as shown in Fig. 1.69, then the work of the force X will be positive. The diagram Δ_F obtained in this way coincides with the line of influence Q_I (Fig. 1. 72).

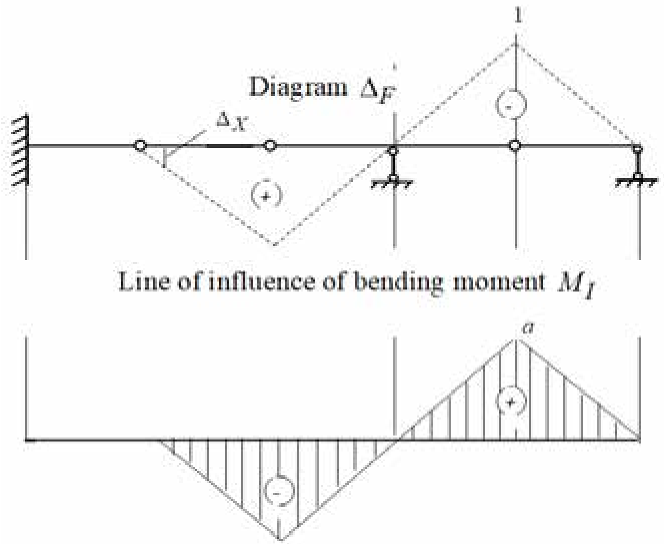


Fig. 1,73. The construction of diagram of influence line of bend moment M_I by kinematic method.

Based on similar considerations, we can construct the line of influence M_I, which is shown in Fig. 1.73.

Self-control questions

1. *What is the way of construction of lines of influence of support reactions of a complex beam according to the general method?*
2. *What is the principle base in the kinematic method of constructing lines of influence based?*
3. *What do we call possible displacements?*

THE EXAMPLES OF INDIVIDUAL WORK
«THE CALCULATION OF COMPLEX BEAM»

The students can get task of this work from table 1 of appendence.

Problem 1.1.

Let us consider the beam, which is shown in Fig. 1.74.

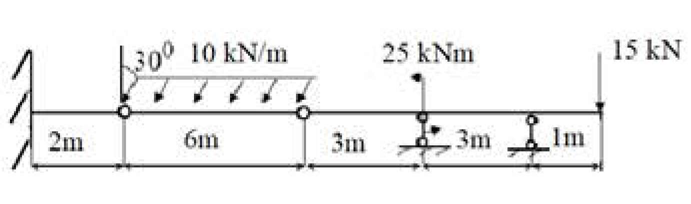


Fig. 1.74. The given beam in Problem 1.1.

We have to run the following actions for a given beam**:**

1. For design scheme of beam to run the kinetic analysis;
2. To define the reactions of beam supports at points A, C, E and F;
3. To check the reaction of supports;
4. To construct the diagrams of internal efforts of given complex beam;

Solution:

1. **Kinematic analysis**.

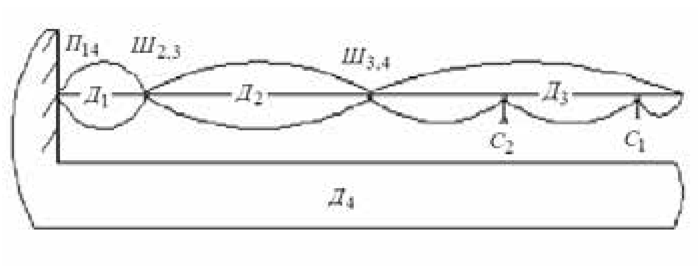


Fig. 1.75. The calculation scheme of a given beam.

1.1. **Quantitative analysis**. The calculation scheme of a given beam consists of four simple disks $Д_1$, $Д_2$, $Д_3$ and $Д_4$. The disk $Д_4$ is the "earth" disk (Fig. 1.75). These disks are connected to each other with the help of two kinematic supports C_1, C_2, two hinges $Ш_{2,3}$, $Ш_{3,4}$, and soldering $П_{14}$.

Thus, according to Chebyshov's formula, we can write:

$$Д = 4, \qquad П = 1, \qquad Ш = 2, \qquad C = 2.$$

Then:

$$Г = 3 \cdot 4 - 3 \cdot 1 - 2 \cdot 2 - 2 - 3 = 12 - 12 = 0.$$

The given beam is statically defined.

1.2. Qualitative analysis.
I stage. (Soldering method)

$$\frac{Д_1 + Д_4}{П_{1,3}} \Rightarrow D_I.$$

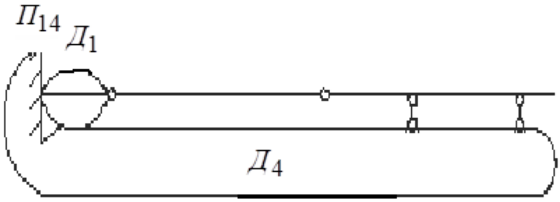


Fig. 1.76. The scheme of first stage of beam installation.

II stage. (Polonso method)

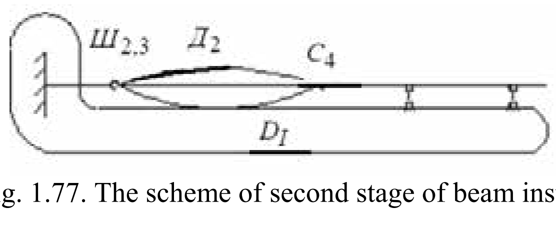


Fig. 1.77. The scheme of second stage of beam installation.

$$\frac{D_I + Д_2}{Ш_{2,3}, C_4} \Rightarrow D_{II}.$$

III stage. (Shukhov's method)

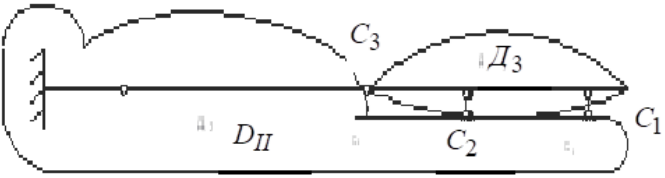


Fig. 1.78. The scheme of third stage of beam installation.

$$\frac{D_{II} + Д_3}{C_1, C_2, C_3} \Rightarrow D_{III}\,.$$

Conclusion: the given structure is geometrically invariable and it built in three stages.

2. Determination of support reactions.

2.1 Let us consider the equilibrium of the disk $Д_2$ (Fig. 1.79):

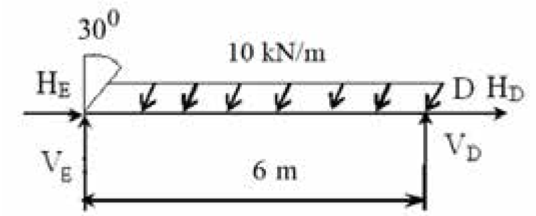


Fig. 1.79. The scheme of equilibrium of the disk $Д_2$.

1. $\sum\limits_{i=1}^{n} F_{ix}^{II} = 0, \qquad H_E = q \cdot 6 \cdot \sin 30^0,$

or

$$H_E = 10 \cdot 6 \cdot 0{,}5 = 30 \text{ kN}.$$

2. $\sum\limits_{i=1}^{n} M_E^{II} = 0, \qquad V_D \cdot 6 - q \cdot 6 \cdot 3 \cdot \cos 30^0 = 0,$

where

$$V_D = q \cdot 3 \cdot \cos 30^0, \quad \text{або} \quad V_D = 10 \cdot 3 \cdot 0{,}866 = 25{,}98 \text{ kN}.$$

3. $\sum_{i=1}^{n} M_D^{II} = 0, \quad -V_E \cdot 6 + q \cdot 6 \cdot 3 \cdot \cos 30^0 = 0,$

where

$$V_E = q \cdot 3 \cdot \cos 30^0, \quad \text{або} \quad V_E = 10 \cdot 3 \cdot 0{,}866 = 25{,}98 \text{ kN}.$$

2.2. Let us consider the equilibrium of the disk $Д_1$ (Fig. 1.80):

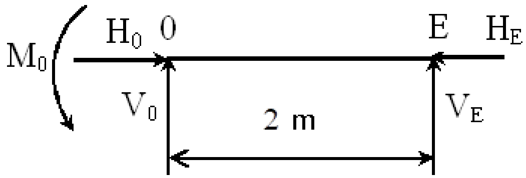


Fig. 1.80. The scheme of equilibrium of the disk $Д_1$.

1. $\sum_{i=1}^{n} F_{ix}^{I} = 0, \quad H_O = H_E = 30$, kN.

2. $\sum_{i=1}^{n} F_{iy}^{I} = 0, \quad V_O = V_E = 25{,}98$ kN.

3. $\sum_{i=1}^{n} M_O^{I} = 0, \quad M_O - V_E \cdot 2 = 0,$

where

$$M_O = V_E \cdot 2, \quad \text{або} \quad M_O = 25{,}98 \cdot 2 = 51{,}06 \text{ kNm}.$$

2.3. From the equilibrium of the disk $Д_3$ (Fig. 1.81), we determine the reactions V_B and V_C:

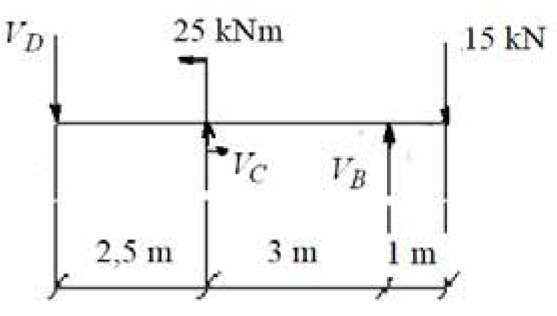


Fig. 1.81. The scheme of equilibrium of the disk $Д_3$.

1. $\sum_{i=1}^{n} F_{ix}^{III} = 0, \quad H_D \equiv 0,$

2. $\sum_{i=1}^{n} M_C^{III} = 0, \quad V_D \cdot 2{,}5 + 25 + V_B \cdot 3 - 15 \cdot 4 = 0,$

where

$$V_B = \frac{-V_D \cdot 2{,}5 - 25 + 15 \cdot 4}{3},$$

or

$$V_B = \frac{-25{,}98 \cdot 2{,}5 - 25 + 15 \cdot 4}{3} = -9{,}98 \text{ kN}.$$

3. $\sum_{i=1}^{n} M_B^{III} = 0, \quad V_D \cdot 5{,}5 + +25 - V_C \cdot 3 - 15 \cdot 1 = 0,$

where

$$V_C = \frac{V_D \cdot 5{,}5 + 25 - 15 \cdot 1}{3},$$

or

$$V_C = \frac{25{,}98 \cdot 5{,}5 + 25 - 15 \cdot 1}{3} = 50{,}96 \text{ kN}.$$

2. Checking of the found support reactions of the given beam (Fig. 1.82):

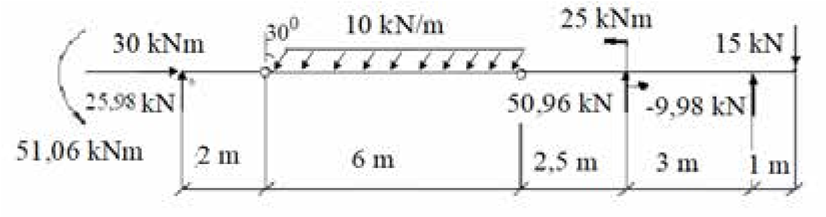


Fig. 1.82. The given beam with support reactions.

1. $\sum_{i=1}^{n} F_{ix} = 0$, $\quad H_O - 10 \cdot 6 \cdot \sin 30^0 = 30 - 60 \cdot 0{,}5 = 30 - 30 = 0$.

2. $\sum_{i=1}^{n} F_{iy} = 0$, $\quad 25{,}98 - 10 \cdot 6 \cdot \cos 30^0 - 51{,}96 - 9{,}98 + 50{,}96 - 15 =$

$$= 16 - 16 = 0.$$

3. $\sum_{i=1}^{n} M_{O_i} = 0$, $\quad 51{,}06 - 15 \cdot 14{,}5 - 9{,}98 \cdot 13{,}5 + 50{,}96 \cdot 10{,}5 + 25 =$

$$= 259{,}81 - 259{,}81 = 0.$$

Therefore, the reactions of the beam supports are determined correctly.

4. Construction of diagrams of internal efforts of the beam.
4.1. Construction of diagram 0f normal forces N_x (Fig. 1.83, b):

I. Portion. $0 \le x_1 \le 2$ m.

$N_{x_1} = 30$ kN.

II. Portion. $2 \le x_2 \le 8$ m.

$N_{x_2} = 30 - q \cdot \sin 30(x_2 - 2)$,

$N_{x_2}(2) = 30$ кН, $N_{x_2}(8) = 30 - 10 \cdot 0{,}5 \cdot 6 = 0$.

III. Portion. $0 \le x_3 \le 1$m.

$N_{x_3} = 0$.

IV. Portion. $1 \le x_4 \le 4$ m.

$N_{x_4} = 0$.

V. Portion. $4 \le x_5 \le 6{,}5$ m.

$N_{x_5} = 0$,

4.2. Construction of diagram of shearing forces Q_x (Fig. 1.83, c):

I. Portion. $0 \le x_1 \le 2$ м.

$Q_{x_1} = 25{,}98$ kN.

II. Portion. $2 \le x_2 \le 8$ m.

$Q_{x_2} = 25{,}98 - q \cdot \cos 30^0 (x_2 - 2)$,

$Q_{x_2}(2) = 25{,}98$ kN, $Q_{x_2}(8) = 25{,}98 - 10 \cdot 0{,}866 \cdot 6 = -25{,}98$ kN.

III. Portion. $0 \le x_3 \le 1$ m.

$Q_{x_3} = 15$ kN.

IV. Portion. $1 \le x_4 \le 4$ m.

$Q_{x_4} = 15 + 9{,}98 = 24{,}98$ kN.

V. Portion. $4 \le x_5 \le 6{,}5$ m.

$Q_{x_5} = 15 + 9{,}98 - 50{,}96 = -25{,}98$ kN.

4.3. Construction of diagram of bending moments M_x (Fig. 1.83, d):

I. Portion. $0 \le x_1 \le 2$ m.

$M_{x_1} = -51{,}96 + 25{,}98 \cdot x_1,$

$M_{x_1}(0) = -51{,}96$ кНм, $M_{x_1}(2) = -51{,}96 + 25{,}98 \cdot 2 = 0$

II. Portion. $2 \le x_2 \le 8$ m.

$$M_{x_2} = -51{,}96 + 25{,}98 \cdot x_1 - q \cdot \cos 30^0 \frac{(x_2 - 2)^2}{2},$$

$M_{x_2}(2) = 0$ kNm,

$M_{x_2}(8) = -51{,}96 + 25{,}98 \cdot 8 - 8{,}66 \cdot 18 = 0.$

$M_{x_2}(5) = -51{,}96 + 25{,}98 \cdot 5 - 8{,}66 \cdot 4{,}5 = 38{,}97$ kNm.

III. Portion. $0 \le x_3 \le 1$ m.

$M_{x_3} = -15 \cdot x_3,$

$M_{x_3}(0) = 0, \qquad M_{x_3}(1) = -15$ kNm.

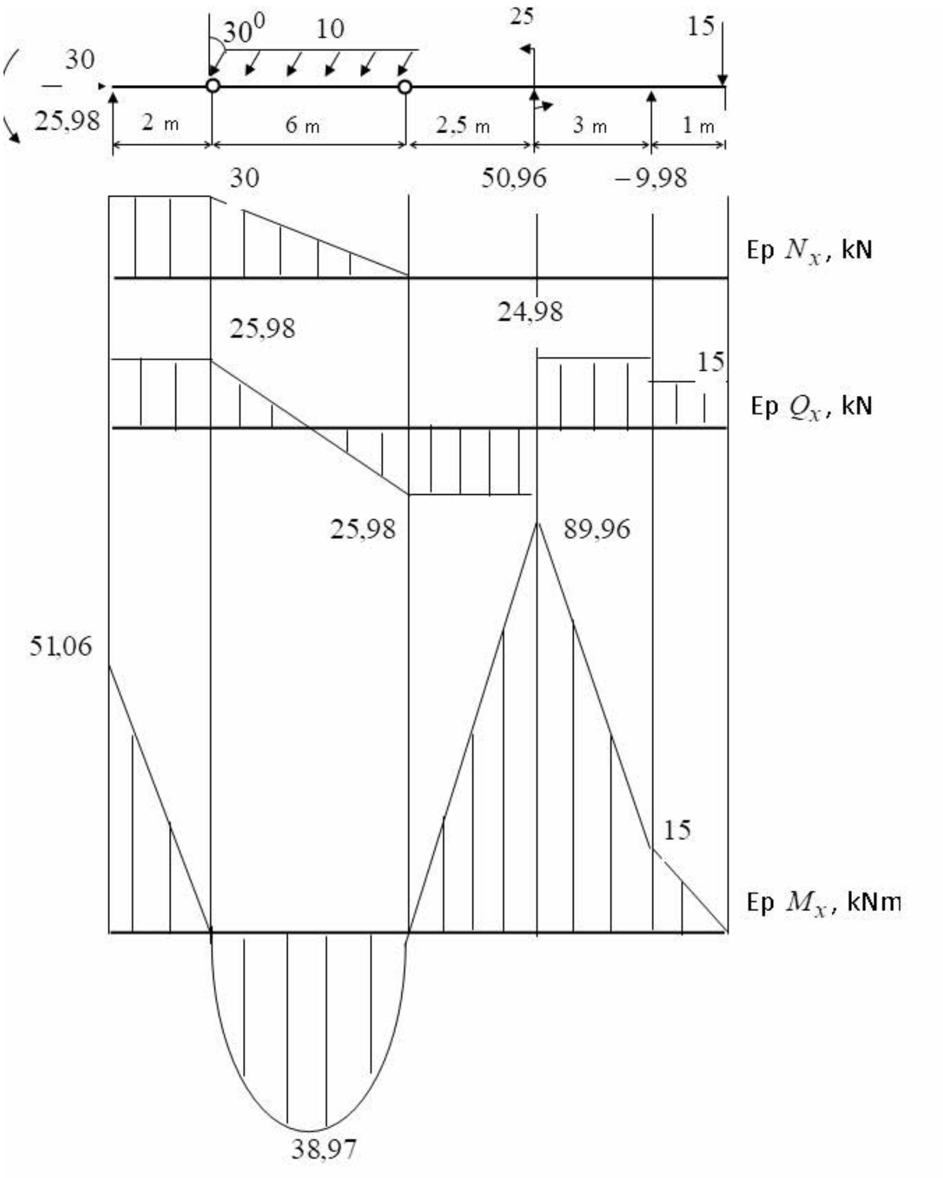


Fig. 1.83. The diagrams of internal efforts for given beam
Problem 1.1.

IV. Portion. $1 \le x_4 \le 4$ m.

$$M_{x_4} = -15 \cdot x_4 - 9{,}98(x_4 - 1),$$

$$M_{x_4}(1) = -15 \text{ kNm},$$

$$M_{x_4}(4) = -15 \cdot 4 - 9{,}98 \cdot 3 = -89{,}96 \text{ kNm}.$$

V. Portion. $4 \le x_5 \le 6{,}5$ m.

$$M_{x_5} = 15 \cdot x_5 + 9{,}98(x_5 - 1) - 50{,}96(x_5 - 4),$$

$$M_{x_5}(6{,}5) = 15 \cdot 6{,}5 + 9{,}98 \cdot 5{,}5 - 50{,}96 \cdot 2{,}5 = -152{,}4 + 152{,}4 = 0.$$

Problem 1.2.
The given complex beam is shown in fig. 1.84.

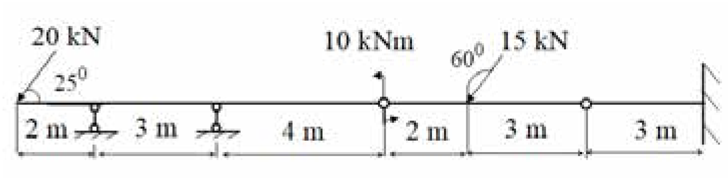


Fig. 1.84. The given beam in Problem 1.2.

We have to run the following actions for a given beam**:**

1. For design scheme of beam to run the kinetic analysis;
2. To define the reactions of beam supports at points A, C, E and F;
3. To check the reaction of supports;
4. To construct the diagrams of internal efforts of given complex beam;

Solution:

1. **Kinematic analysis**.

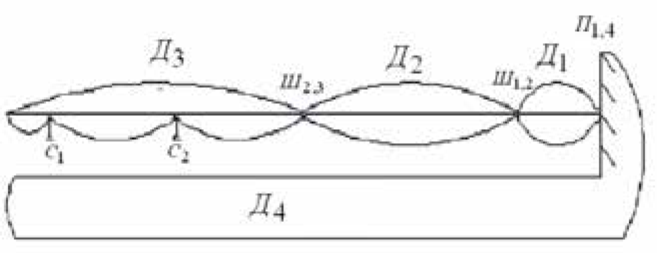


Fig. 1.85. The calculation scheme of a given beam.

1.1. **Quantitative analysis**.

The calculation scheme of a given beam consists of four simple disks $Д_1$, $Д_2$, $Д_3$ and $Д_4$. The disk $Д_4$ is the "earth" disk (Fig. 1.82). These disks are connected to each other with the help of two kinematic supports C_1, C_2, two hinges $Ш_{2,3}$, $Ш_{3,4}$, and soldering $П_{14}$.

Thus, according to Chebyshov's formula, we can write:

$$Д = 4, \qquad П = 1, \qquad Ш = 2, \qquad C = 2.$$

Then:

$$Г = 3 \cdot 4 - 3 \cdot 1 - 2 \cdot 2 - 2 - 3 = 12 - 12 = 0.$$

The given beam is statically defined.

1.2. Qualitative analysis.

I stage. (Soldering method)

$$\frac{Д_1 + Д_4}{П_{1,3}} \Rightarrow D_I .$$

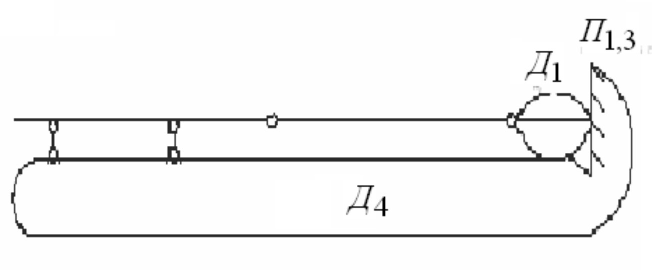
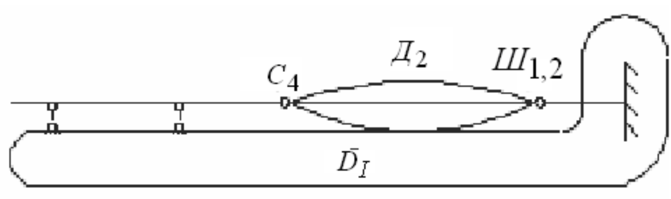


Fig. 1.86. The scheme of first stage of beam installation.

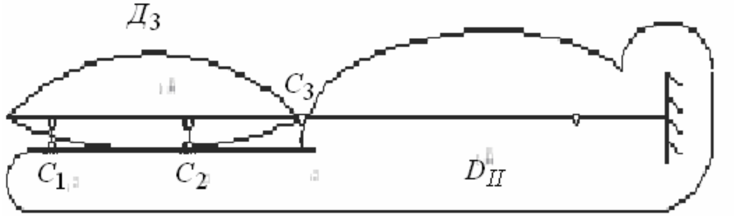
II stage. (Polonso method)

$$\frac{D_I + Д_2}{Ш_{1,2}, C_4} \Rightarrow D_{II} .$$



1.87. The scheme of second stage of beam installation.

III stage. (Shukhov's method)



1.88. The scheme of third stage of beam installation.

$$\frac{D_{II} + Д_3}{C_1, C_2, C_3} \Rightarrow D_{III}.$$

Conclusion: the given structure is geometrically invariable and it built in three stages.

2. **Determination of support reactions.**

2.1 Let us consider the equilibrium of the disk $Д_2$ (Fig. 1.89):

1. $\sum\limits_{i=1}^{n} F_{ix}^{II} = 0,\qquad H_D - H_C - 15 \cdot \sin 60^0 = 0,$

or

$$H_D = H_C + 15 \cdot 0{,}866 = H_C + 12{,}99.$$

2. $\sum\limits_{i=1}^{n} M_C^{II} = 0,\qquad V_D \cdot 5 - 15 \cdot 2 \cdot \cos 60^0 + 10 = 0,$

where

$$V_D = 6 \cdot \cos 60^0 - 2, \text{ або } V_D = 6 \cdot 0{,}5 - 2 = 1 \text{ kN}.$$

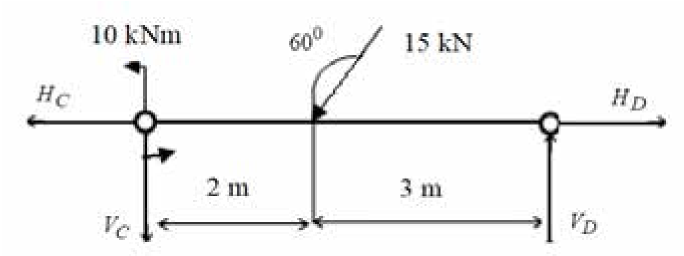


Fig. 1.89. The scheme of equilibrium of the disk $Д_2$.

3. $\sum\limits_{i=1}^{n} M_D^{II} = 0,\qquad V_C \cdot 5 + 15 \cdot 3 \cdot \cos 60^0 + 10 = 0,$

where

$$V_C = -9 \cdot \cos 60^0 - 2, \text{ або } V_E = -9 \cdot 0{,}5 - 2 = -6{,}5 \text{ кN}.$$

2.2. Let us consider the equilibrium of the disk $Д_1$ (Fig. 1.90):

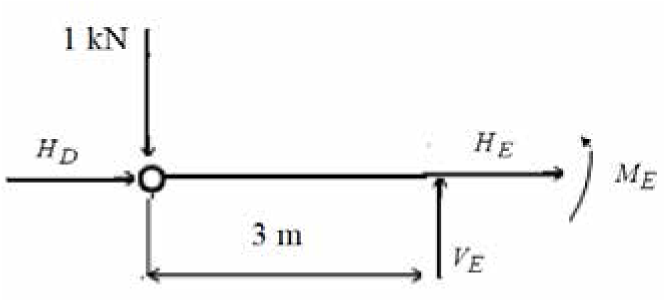


Fig. 1.90. The scheme of equilibrium of the disk.

1. $\sum\limits_{i=1}^{n} F_{ix}^{I} = 0, \qquad H_E = H_D.$

2. $\sum\limits_{i=1}^{n} F_{iy}^{I} = 0, \qquad V_E = 1$ kN.

3. $\sum\limits_{i=1}^{n} M_E^{I} = 0, \qquad M_E + 1 \cdot 3 = 0,$

Finally, we obtain that:

$$M_E = -3 \text{ kNm}.$$

2.3. From the equilibrium of the disk $Д_3$ (Fig. 1.91), we determine the reaction of supports R_B and R_A:

1. $\sum\limits_{i=1}^{n} F_{ix}^{III} = 0, \qquad H_C - 20 \cdot \cos 25^0 = 0,$

where

$H_C = 20 \cdot \cos 25^0, \qquad$ або $\quad H_C = 20 \cdot 0{,}9063 = 18{,}126$ kN.

Finally, we get:

$H_D = 18{,}126 + 12{,}99 = 31{,}116$ kN,

$H_E = H_D = 31{,}116$ kN.

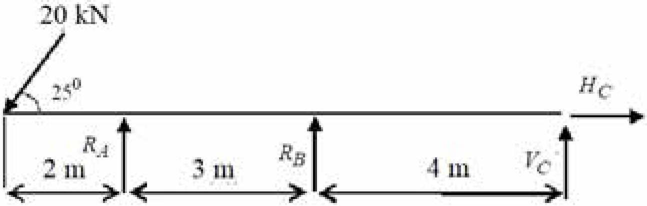


Fig. 1.91. The scheme of equilibrium of the disk $Д_3$.

2. $\sum_{i=1}^{n} M_B^{III} = 0, \quad -R_A \cdot 3 + 20 \cdot 5 \cdot \sin 25^0 + V_C \cdot 4 = 0,$

where

$$R_A = \frac{V_c \cdot 4 + 100 \cdot \sin 25^0}{3},$$

or

$$R_A = \frac{-6{,}5 \cdot 4 + 100 \cdot 0{,}4226}{3} = 5{,}42 \text{ kN}.$$

3. $\sum_{i=1}^{n} M_A^{III} = 0, \quad R_B \cdot 3 + 20 \cdot 2 \cdot \sin 25^0 + V_C \cdot 7 = 0,$

where

$$R_B = \frac{-V_C \cdot 7 - 40 \cdot \sin 25^0}{3},$$

or

$$R_B = \frac{6{,}5 \cdot 7 - 40 \cdot 0{,}4226}{3} = 9{,}532 \text{ kN}.$$

3. Let us check the found support reactions of the given beam (Fig. 1.92):

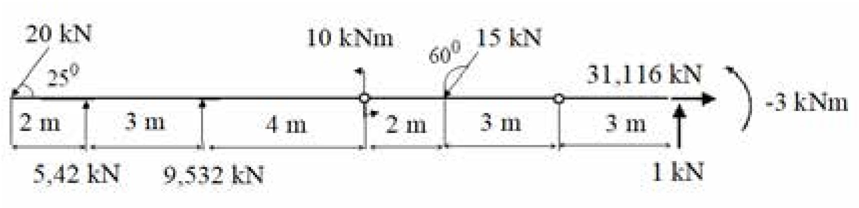


Fig. 1.82. The given beam with support reactions.

1. $\sum\limits_{i=1}^{n} F_{ix} = 0$, $\quad 31{,}116 - 15 \cdot \sin 60^0 - 20 \cdot \sin 25^o = .$

$$= 31{,}116 - 15 \cdot 0{,}866 - 20 \cdot 0{,}963 = 31{,}116 - 31{,}116 = 0 .$$

2. $\sum\limits_{i=1}^{n} F_{iy} = 0$, $\quad -20 \cdot 0{,}4226 - 15 \cdot 0{,}5 + 5{,}42 + 9{,}532 + 1 =$

$$= 15{,}942 - 15{,}942 = 0 .$$

3. $\sum\limits_{i=1}^{n} M_{O_i} = 0$, $\quad 5{,}42 \cdot 2 - 15 \cdot 0{,}5 \cdot 11 + 9{,}532 \cdot 5 + 10 + 1 \cdot 17 - 3 =$

$$= 85{,}5 - 85{,}5 = 0 .$$

Therefore, the reactions of the beam supports are determined correctly.

4. Construction of diagrams of internal efforts of the beam.

4.1. Construction of diagram of normal forces N_x (Fig. 1.93):

I. Portion. $0 \le x_1 \le 2$ m.

$N_{x_1} = -20 \cdot 0{,}9063 = -18{,}126$ kN.

II. Portion. $2 \le x_2 \le 5$ m.

$N_{x_2} = -18{,}126$ kN.

III. Portion. $5 \le x_3 \le 9$ m.

$N_{x_3} = -18{,}126$ kN.

IV. Portion. $0 \le x_4 \le 3$ m.

$N_{x_4} = -31{,}116$ kN.

V. Portion. $3 \le x_5 \le 6$ m.

$N_{x_5} = -31{,}116$ kN.

VI. Portion. $6 \le x_5 \le 8$ m.

$N_{x_6} = -31{,}116 + 15{\cdot},866 = -18{,}126$ kN.

4.2. Construction of diagram of shearing forces Q_x (Fig. 1.93):

I. Portion. $0 \le x_1 \le 2$ m.

$Q_{x_1} = -20 \cdot \sin 25^0$,

or

$Q_{x_1} = -20 \cdot 0{,}4226 = -8{,}452$ kN.

II. Portion. $2 \le x_2 \le 5$ m.

$Q_{x_2} = -8{,}452 + 5{,}42 = -3{,}32$ kN.

III. Portion. $5 \le x_3 \le 9$ m.

$Q_{x_3} = -8{,}452 + 5{,}42 + 9{,}536 = 6{,}5$ kN.

IV. Portion. $0 \le x_4 \le 3$ m.

$Q_{x_4} = -1$ kN.

V. Portion. $3 \le x_5 \le 6$ m.

$Q_{x_5} = -1$ kN.

VI. Portion. $6 \le x_5 \le 8$ m.

$Q_{x_6} = -1 + 15 \cdot \cos 60^0$

or
$Q_{x_6} = -1 + 15 \cdot 0{,}5 = -6{,}5$ kN.

4.3. Construction of diagram of bending moments M_x (Fig. 1.93):

I. Portion. $0 \le x_1 \le 2$ m.

$M_{x_1} = -8{,}452{,}\cdot x_1$,

$M_{x_1}(0) = 0$, $M_{x_1}(2) = -8{,}452 \cdot 2 = -16{,}904$ kNm.

II. Portion. $2 \le x_2 \le 5$ m.

$M_{x_2} = -8{,}452 \cdot x_2 + 5{,}42 \cdot (x_2 - 2)$,

$M_{x_2}(2) = -16{,}904$ kNm,

$M_{x_2}(5) = -8{,}452 \cdot 5 + 5{,}42 \cdot 3 = 26$ kNm.

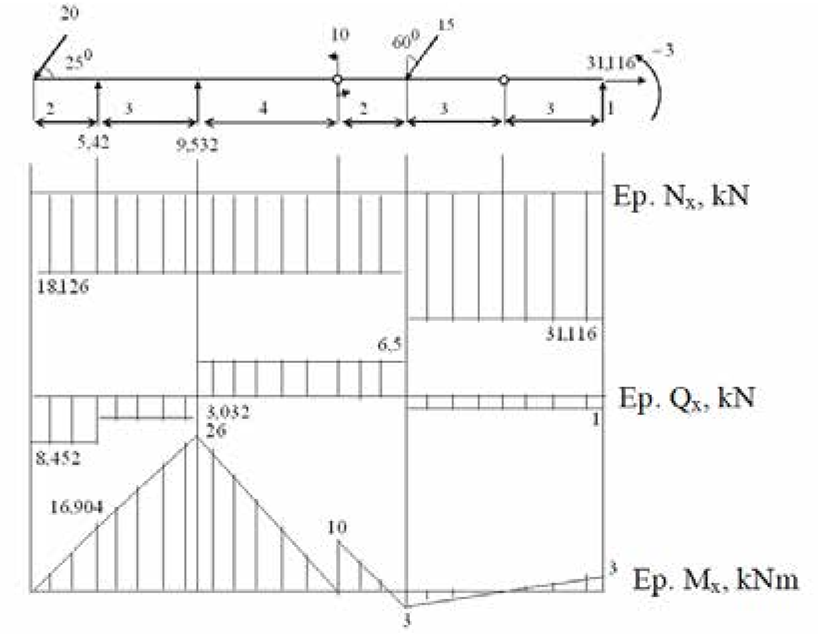


Fig. 1.93. The diagrams of internal efforts for given beam in Problem 2.

III. Portion. $5 \le x_3 \le 9$ m.

$$M_{x_3} = -8{,}452 \cdot x_3 + 5{,}42 \cdot (x_3 - 2) + 9{,}532 \cdot (x_3 - 5),$$

$M_{x_3}(5) = 26$ kNm, $\quad M_{x_3}(9) = 0.$

IV. Portion. $0 \le x_4 \le 3$ m.

$$M_{x_4} = -3 - 1 \cdot x_4,$$

$M_{x_4}(0) = -3$ kNm, $\quad M_{x_4}(3) = 0$.

V. Portion. $3 \le x_5 \le 6$ m.

$M_{x_5} = -3 - 1 \cdot x_5$,
$M_{x_5}(6) = -3 + 1 \cdot 6 = 3$ kNm.

VI. Portion. $6 \le x_5 \le 8$ m.

$M_{x_6} = -3 + 1 \cdot x_6 - 15 \cdot \cos 60^0 (x_6 - 6)$
or
$M_{x_6}(8) = -3 + 1 \cdot 8 - 15 \cdot 0{,}5 \cdot 2 = -10$ kNm.

Problem 1.3.
Let a complex beam is loaded with an external load according to fig. 1.94.

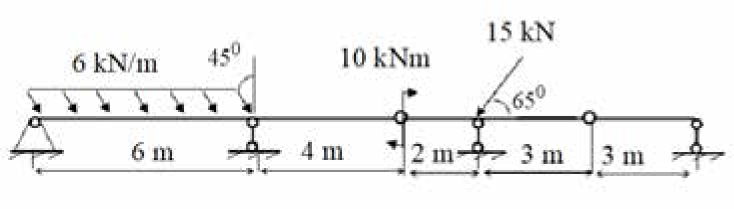


Fig. 1.94. The given beam in Problem 1.3.

We have to run the following actions for a given beam**:**
1. For design scheme of beam to run the kinetic analysis;
2. To define the reactions of beam supports at points A, C, E and F;
3. To check the reaction of supports;
4. To construct the diagrams of internal efforts of given complex beam;

Solution:

1. **Kinematic analysis**.

1.1. Quantitative analysis. The calculation scheme of a given beam consists of four simple disks $Д_1$, $Д_2$, $Д_3$ and $Д_4$. The disk $Д_4$ is the "earth" disk (Fig. 1.95). These disks are connected to each other with the help of three kinematic supports C_1, C_2, C_3 and three hinges $Ш_{1,4}$, $Ш_{2,3}$, $Ш_{3,4}$.

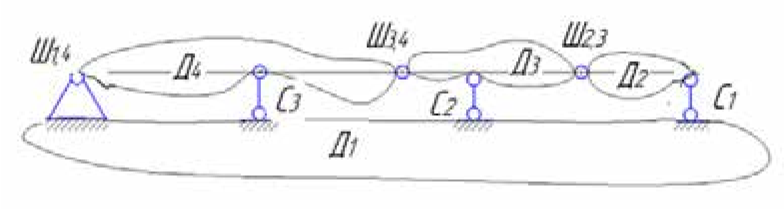


Fig. 1.95. The calculation scheme of a given beam.

Thus, according to Chebyshov's formula, we can write:

$$Д = 4, \qquad Ш = 3, \qquad C = 3.$$

Where we get:

$$Г = 3 \cdot 4 - 2 \cdot 3 - 3 - 3 = 12 - 12 = 0.$$

Thus, the given beam is statically defined.

1.2. Qualitative analysis.

I. stage (Polonso's method). At the first stage of installation, the disks $Д_2$ and $Д_1$ are connected using a kinematic support C_1 and a hinge and $Ш_{2,3}$ a invariable disk D_I is formed (Fig. 1.96).

Fig. 1.96. The scheme of first stage of beam installation.

$$\frac{Д_1 + Д_2}{C_1, Ш_{2,3}} \Rightarrow D_I$$

II stage (Polonso's method). At the second stage of installation, the disks D_I and $Д_3$ are connected by using a kinematic support C_2 and a hinge $Ш_{3,4}$ (Fig. 1.97) and an invariable disk D_{II} is formed:

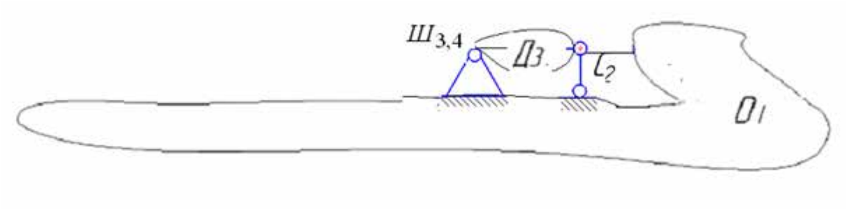


Fig. 1.97. The scheme of second stage of beam installation.

$$\frac{D_I + Д_3}{C_2, Ш_{3,4}} \Rightarrow D_{II} .$$

III stage (Polonso's method). At the third stage of installation of system, the disks D_{II} and $Д_4$ are connected by using a kinematic support C_3 and a hinge $Ш_{1,4}$ (Fig. 1.98) and an invariable disk D_{III} is formed:

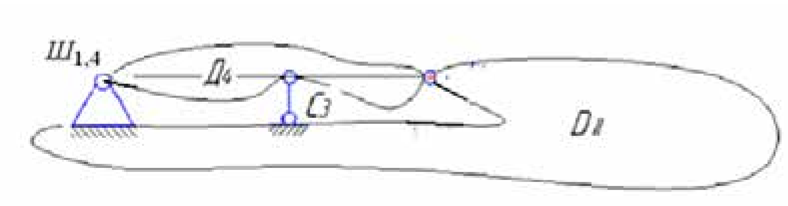


Fig. 1.98. The scheme of third stage of beam installation.

$$\frac{D_{II} + Д_4}{C_3, Ш_{1,4}} \Rightarrow D_{III} .$$

The given construction is geometrically invaraible and it is built in 3 stages.

2. Determination of support reactions of the beam.

2.1. Consider the equilibrium of the disk (Fig. 1.99)**:**

1. $\sum_{i=1}^{n} F_{ix}^{II} = 0, \qquad H_E = 0.$

2. $\sum_{i=1}^{n} M_E^{II} = 0, \qquad V_F \cdot 3 = 0,$

where

$$V_F = 0.$$

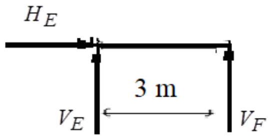


Fig. 1.99. The equilibrium of the disk $Д_2$.

3. $\sum_{i=1}^{n} M_F^{II} = 0, \qquad -V_B \cdot 3 = 0,$

where

$$V_B = 0.$$

2.2. Let us consider the equilibrium of the disk $Д_3$ (Fig. 1.100):

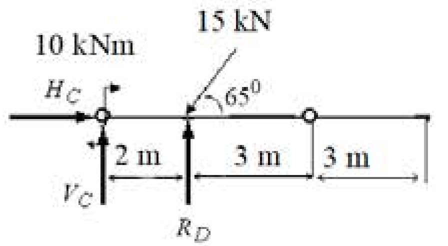


Fig. 1.100. The equilibrium of the disk $Д_3$.

1. $\sum_{i=1}^{n} F_{ix}^{III} = 0, \qquad H_C - 15 \cdot \cos 65^0 = 0,$

where

$$H_C = 15 \cdot \cos 65^0, \qquad \text{або} \quad H_C = 15 \cdot 0{,}4226 = 6{,}34 \text{ kN.}$$

2. $\sum_{i=1}^{n} M_D^{III} = 0, \quad -10 - V_C \cdot 2 = 0,$

or

$$V_C \cdot = -5 \text{ kN.}$$

3. $\sum_{i=1}^{n} M_C^{III} = 0, \quad -10 + R_D \cdot 2 - 15 \cdot 2 \cdot \sin 65^0 = 0,$

where

$$R_D = 5 + 15 \cdot \sin 65^0 = 0$$

or

$$R_D \cdot = 5 + 15 \cdot 0{,}9063 = 18{,}595 \text{ kN.}$$

2.3. Let us consider the equilibrium of the disk $Д_4$ (Fig. 1.101):

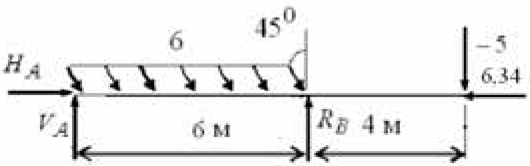


Fig. 1.101. The equilibrium of the disk $Д_4$.

1. $\sum_{i=1}^{n} F_{ix}^{IV} = 0, \qquad H_A + 6 \cdot 6 \cdot \sin 45^0 - 6{,}34 = 0,$

or

$$H_A = 6{,}34 - 36 \cdot \sin 45^0 = -19{,}112 \text{ kN}.$$

2. $\sum_{i=1}^{n} M_{A_i}^{IV} = 0$, $\quad 5 \cdot 10 + R_B \cdot 6 - 6 \cdot 6 \cdot 3 \cdot \cos 45^0 = 0$,

where

$$R_B = \frac{108 \cdot \cos 45^0 - 50}{6}$$

or

$$R_B = 4{,}393 \text{ kN}.$$

3. $\sum_{i=1}^{n} M_{E_i}^{IV} = 0$, $\quad 5 \cdot 4 - V_A \cdot 6 + 6 \cdot 6 \cdot 3 \cdot \cos 45^0 = 0$,

where

$$V_A = \frac{108 \cdot \cos 45^0 + 20}{6}$$

or

$$V_A = 16{,}059 \text{ kN}.$$

3. We run general check of the found reactions of the beam. For this, we apply the found reactions to the given beam (Fig. 1.102).

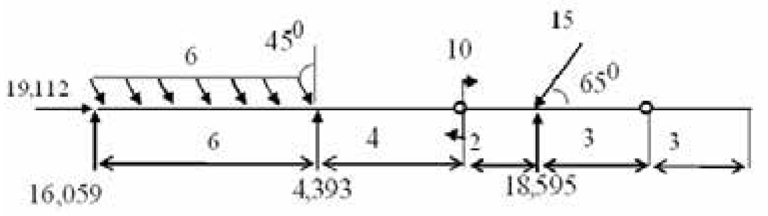


Fig. 1.102. The given beam with support reactions.

1. $\sum_{i=1}^{n} F_{ix} = 0$, $\quad -19{,}112 + 6 \cdot 6 \cdot 0{,}707 - 15 \cdot 0{,}4226 =$

$$= 25{,}452 - 19{,}112 - 6{,}34 = 25{,}452 - 25{,}452 = 0.$$

2. $\sum_{i=1}^{n} F_{iy} = 0$, $\quad 16{,}059 + 4{,}393 + 18{,}595 - 15 \cdot 0{,}9063 - 6 \cdot 6 \cdot 0{,}707 =$

$$= 16{,}059 + 4{,}393 + 18{,}595 - 13{,}595 - 25{,}452 =$$
$$= 39{,}047 - 39{,}047 = 0.$$

3. $\sum_{i=1}^{n} M_{D_i} = 0$, $\quad -16{,}059 \cdot 12 - 10 - 4{,}393 \cdot 6 + 6 \cdot 6 \cdot 9 \cdot 0{,}707 =$

$$= -192{,}708 - 10 - 26{,}358 + 229{,}068 =$$
$$-229{,}07 + 229{,}07 = 0.$$

Therefore, the reactions of the beam supports are determined correctly.

4. We will build the diagrams of internal efforts for a given beam.
4.1. Construction of diagram of normal force N_x (Fig. 1.103):

I. Portion. $0 \le x_1 \le 6$ m.

$$N_{x_1} = -19{,}112 + 6 \cdot 0{,}707 \cdot x_1,$$
or
$$N_{x_1} = -19{,}112 + 4{,}242 \cdot x_1.$$

Then

$$N_{x_1}(0) = -19{,}112 \text{ kN}, \; N_{x_1}(6) = 6{,}34 \text{ kN}.$$

II. Portion. $0 \le x_2 \le 3$ m.

$$N_{x_2} = 0.$$

III. Portion. $3 \le x_3 \le 6$ m.

$$N_{x_3} = 0.$$

IV. Portion. $6 \le x_4 \le 8$ m.

$N_{x_4} = 15 \cdot 0{,}4226 = 6{,}34$ kN.

V. Portion. $8 \le x_5 \le 12$ m.

$N_{x_5} = 6{,}34$ kN.

4.2. The construction of the diagram of shearing forces Q_x (Fig. 1.103):

I. Portion. $0 \le x_1 \le 6$ m.

$Q_{x_1} = 16{,}059 - 6 \cdot 0{,}707 \cdot x_1$.
or

$Q_{x_1} = 16{,}059 - 4{,}242 \cdot x_1$.

Then

$Q_{x_1}(0) = 6{,}059$ kN, $Q_{x_1}(6) = -9{,}393$ kN.

II. Portion. $0 \le x_2 \le 3$ m.

$Q_{x_2} = 0$.

III. Portion. $3 \le x_3 \le 6$ m.

$Q_{x_3} = 0$.

IV. Portion. $6 \le x_4 \le 8$ m.

$Q_{x_4} = -18{,}595 + 15 \cdot 0{,}9063 = -5$ kN.

V. Portion. $8 \le x_5 \le 12$ m.

$Q_{x_5} = -5$ kN.

4.3. The construction of the diagram of bending moments M_x (Fig. 1.103):

I. Portion. $0 \le x_1 \le 6$ m.

$$M_{x_1} = 16{,}059 \cdot x_1 - 4{,}242 \cdot \frac{x_1^2}{2} = 16{,}059 \cdot x_1 - 2{,}121 \cdot x_1^2;$$

We find the location of the top of the parabola:

$$\frac{dM_{x_1}}{dx_1} = 16{,}059 - 4{,}242 \cdot x_1 = 0,$$

where

$$x_1 = \frac{16{,}059}{4{,}242} \approx 3{,}8 \text{ m}.$$

Then:

$M_{x_1}(0) = 0;$ $M_{x_1}(3{,}8) = 30{,}4$ kNm, $M_{x_1}(6) = 20$ kNm.

II. Portion. $0 \le x_2 \le 3$ m.

$M_{x_2} \equiv 0$.

III. Portion. $3 \le x_3 \le 6$ m.

$M_{x_3} \equiv 0$.

IV. Portion. $6 \le x_4 \le 8$ m.

$M_{x_4} = 5 \cdot (x_4 - 6).$

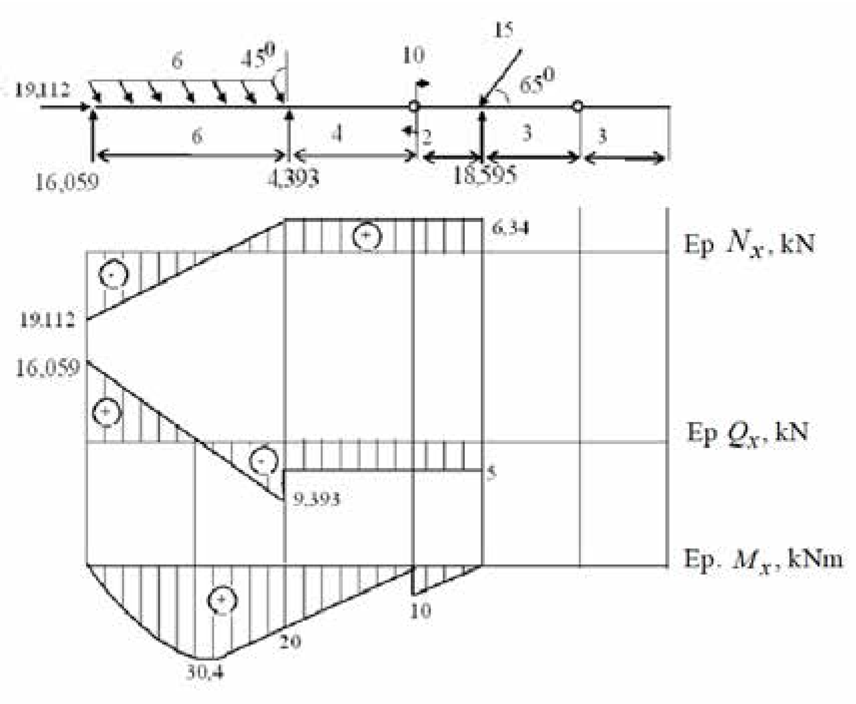


Fig. 1.103. The diagrams of internal efforts for given beam in Problem 1.3.

Then:

$M_{x_4}(6) = 0; \qquad M_{x_4}(8) = 10$ kNm.

V. Portion. $8 \le x_5 \le 12$ m.

$M_{x_5} = -10 + 5 \cdot (x_5 - 6),$

Then:

$M_{x_5}(8) = 0; \qquad M_{x_5}(12) = 20$ kNm.

CHAPTER II

TRUSS CALCULATIONS

From the point of view of material using, when spans of solid beams and columns were increased, they are transformed into cross-lattice structures, which are more rational contracture. Under calculation trusses can model these structures.

Definition 2.1. We will call trusses geometrically invariable calculation models formed from rectilinear bars, which are connected to each other at the extreme points in joint, which are loaded in the form of nodal concentrated forces.

Trusses in rural construction are most often found in attic buildings like as roof structures (Fig. 2.1).

Fig. 2.1. Using of wood trusses like as roof structures.

Not often, we can find the use of trusses as floor constructions (Fig. 2.2)

Fig. 2.2. The installation of trusses like as floor constructions.

Based on the first axiom of statics, only normal force arises in the elements of such a calculation scheme (unloaded simple disks with two hinged connections to other ones). Such an idealization is possible due to the great flexibility of the bars of real lattice structures and the constructive solution of transferring the payload to the places where individual bars are combined. The adopted idealization significantly simplifies the calculation of this class of structures without any significant loss in the reliability of the obtained results.

THEME 1.
THE METHOD OF JOINTS

We will consider only flat trusses. Since spatial lattice structures are reduced to flat models easily, all elements of these trusses belong to the same plane in which the load on the truss acts.

The main elements of a flat truss (Fig. 2.3) are belts — bars placed on the outer contour (CB — upper belt and AB — lower belt).The crate is a set of bars that connect in the belts. Vertical bar crates are called racks (compressed) or suspensions (stretched).

The distance L between the axes of the supports of the truss is called its span, and the horizontal distance between adjacent joints of the upper or lower belts is called the truss panel. The overall size of the truss between the belts is its height h.

Trusses are classified by the geometry of the belts, by the type of crate, by the method of support, by purpose and by the level of traffic on it.

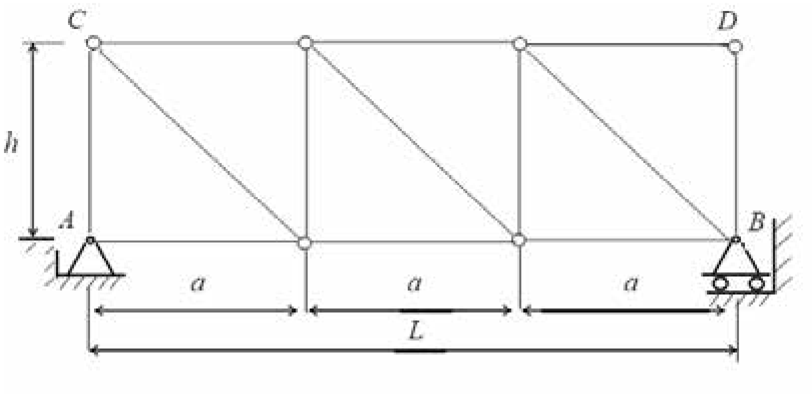


Fig. 2.3. The design scheme of truss.

The calculation method of the truss is significantly influenced by its place in the classification according to the structural scheme: beam (span or cantilever), arched (spanning), and according to the type of crate: triangular, diagonal. semi-braced, multi-braced, rhombic lattice, etc. The calculation of the truss consists in determining the normal forces in all its bars, which will later be used in the selection (checking) of the cross section of its elements. There are analytical and graphical methods for calculating flat statically determined trusses. The graphic method is less accurate, so we will not dwell on it.

The simplest statically determined trusses are formed from a hinged triangle by sequentially connecting nodes with the help of two non-parallel bars (Dyad method). At the last stage of creating the calculation model, the constructed disk-truss is connected to the "ground" disk using Polonzo's method or Shukhov's method. The method of sections is used to determine the internal forces in the bars of such joint-bar models. Depending on the type of section, the method of sections and the method of joints are distinguished.

According to method of joints, one node can be separated from the truss by an imaginary section (Fig. 2.4), and by the method of sections - an arbitrary fragment of the truss. In the first case, the system of forces acting on one node is convergent (Fig. 2.5). The equilibrium condition of given node is written by two equations of force projections on the coordinate axes x and y. Two unknown values of internal forces can be determined from the two equations, for example as in Fig. 2.5 is effort N_7 and N_8.

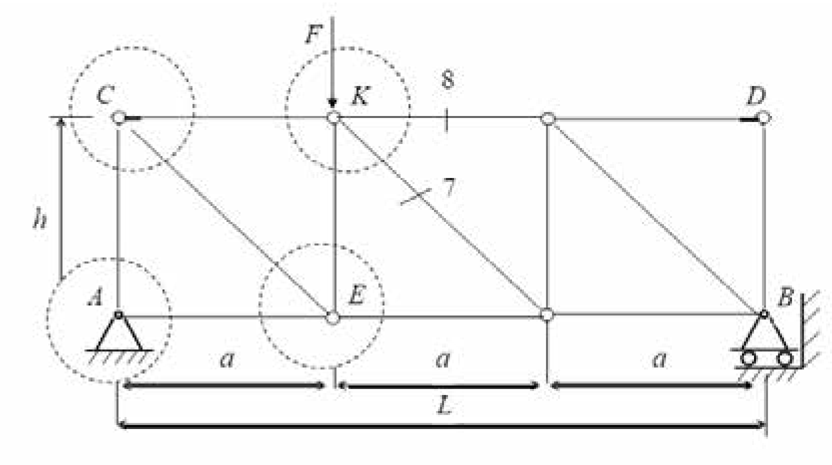


Fig. 2.4. The context of method of joints.

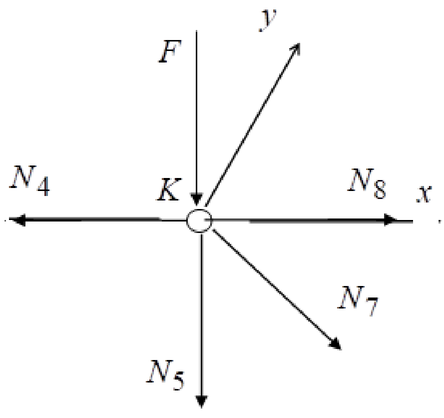


Fig. 2.5. The example of cut out joint K of truss.

Method of joints is based on the algorithm of sequential cutting of them. In consider joint only two bars with unknown forces are "cut" each time. The values of normal forces are determined by the method of projections. The simplicity of calculation of the values of the internal forces in the bars is achieved by successive projection of the convergent system of forces on the axis perpendicular to each of the two unknowns (axes x and y). The forces in other "cut" bars, which are applied to node, must be given. According to this algorithm, it is impossible to determine the internal force in any bar at once - it must be "reached" by successively cutting a set of joints, starting with the two-bars (joints A, C, E are shown in Fig. 2.4).

Let us consider an example of the calculation of the truss, which is shown in Fig. 2.6.

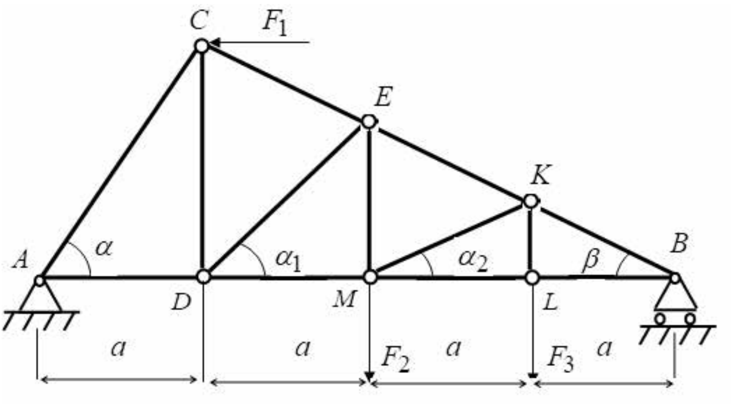


Fig. 2.6. The triangle truss.

According to Fig. 2.6 the following loads are applied to the joints of the truss: $F_1 = 20$kN, $F_2 = 50$ kN, $F_3 = 70$ kN.

We are going to determine by the method of joints the normal forces in the bars of the given truss, using its geometric dimensions and taking into account that: $\alpha = 55^0$.

1. Kinematic analysis of the calculation scheme. We number the bars of the given truss. Using Fig. 2.7, and we can write for the Chebyshov's formula:

$$Д = 1; \qquad В = 8; \qquad Ш = 1; \qquad С = 14.$$

Let us write down Chebyshov's formula for the given calculation scheme:

$$\Gamma = 3 \cdot 1 + 8 \cdot 2 - 2 \cdot 1 - 14 - 3 = 19 - 19 = 0.$$

Thus, the given truss is a statically definitely system.

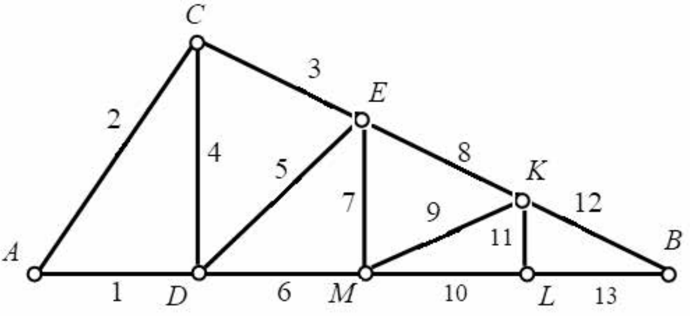


Fig. 2.7. The calculation scheme of triangle truss.

In addition, the given calculation scheme is construct in nine assembly stages, the first eight of which are run, using the "dyad" method, and the last one is the Polonzo’s method.

Thus, we finally conclude that the given flat truss is a statically definitely and geometrically invariable system.

2. We remove the hinge and the kinematic support and have to apply the appropriate reactions. Since the support is a fixed by cylindrical joint, because it will be represent by two components: horizontal and vertical. The support is a kinematic supports has only vertical reaction. We denote the positive directions of the axes x and y (Fig. 2.8).

First, we will run the geometric refinements.

$tg\alpha = tg55^0 = 1{,}42815$; $\sin\alpha = 0{,}819152$;

$\cos\alpha = 0{,}573576$.

Where: $CD = a \cdot tg\alpha = a \cdot 1{,}42815$.

$tg\beta = \frac{CD}{3a} = 0{,}476049$. Then: $\beta = 25{,}4568^0$.

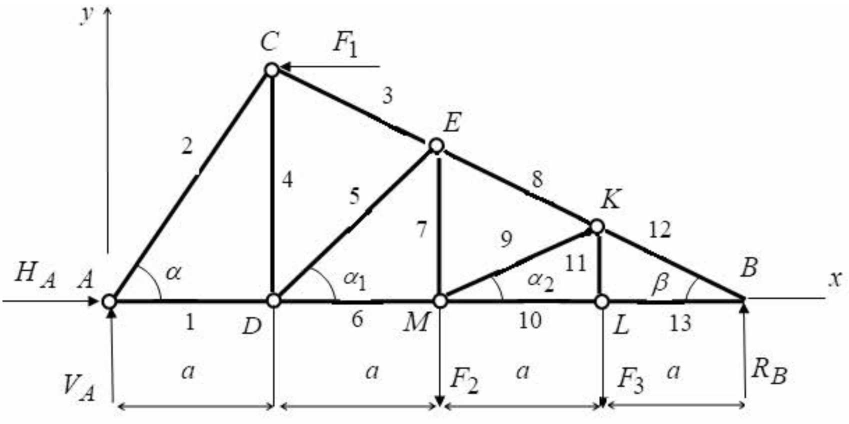


Fig. 2.8. The triangle truss with reaction of supports.

We get that:

$\sin\beta = 0{,}429830$; $\cos\beta = 0{,}902910$.

$tg\alpha_1 = \frac{2}{3}\frac{CD}{a} = 0{,}9521$. Where: $\alpha_1 = 43{,}5944^0$.

We get that:

$\sin\alpha_1 = 0{,}689548$; $\cos\alpha_1 = 0{,}724240$.

$tg\alpha_2 = \frac{1}{3}\frac{CD}{a} = 0{,}47605$,   where: $\alpha_2 = 25{,}4568^0$.

Then we get that:

$\sin\alpha_2 = 0{,}429830$; $\cos\alpha_2 = 0{,}902910$.

Let us start the calculation of the truss by determining the reactions of the supports. So, taking into account three external forces: F_1, F_2, F_3, we are going to write three equations of equilibrium of the truss:

1. $\sum\limits_{i=1}^{n} F_{ix} = 0;\qquad H_A - F_1 = 0.$

2. $\sum\limits_{i=1}^{n} M_A(F_i) = 0;\qquad F_1 \cdot CD - F_2 \cdot 2a - F_3 \cdot 3a + R_B \cdot 4a = 0.$

3. $\sum\limits_{i=1}^{n} F_{iy} = 0;\qquad V_A - F_3 - F_2 + R_B = 0.$

From these equations we find:

$$H_A = F_1;$$

$$R_B \cdot = \frac{1}{4}\left(F_1 \cdot \frac{CD}{a} - 2F_2 - 3F_3 \right) = 0{,}5F_2 + 0{,}75F_3 - 0{,}3570375F_1;$$

$$V_A = F_3 + F_2 - R_B = 0{,}5F_2 + 0{,}25F_3 + 0{,}3570375F_1.$$

Substituting the values of the forces F_1, F_2, F_3, into the obtained expressions for the reactions, we finally find:

$$H_A = 20 \text{ kN};\quad R_B \cdot = 70{,}36 \text{ kN};\qquad V_A = 49{,}64 \text{ kN}.$$

3. Let us to determine the forces in the truss bars by the method of joints. Since in this case it is a flat truss, it is necessary to start from a node to which no more than two bars with unknown forces converge.

Consider the equilibrium of node A. Firstly, we apply to this node known forces H_A and V_A. Then we apply unknown (by magnitude) forces N_1 and N_2 (Fig. 2.9). We write down two equilibrium conditions of a plane convergent system of forces H_A, V_A, N_1, N_2:

1. $\sum_{i=1}^{n} F_{ix} = 0;$ $\quad H_A + N_1 + N_2 \cos\alpha = 0.$

2. $\sum_{i=1}^{n} F_{iy} = 0;$ $\quad V_A + N_2 \sin\alpha = 0.$

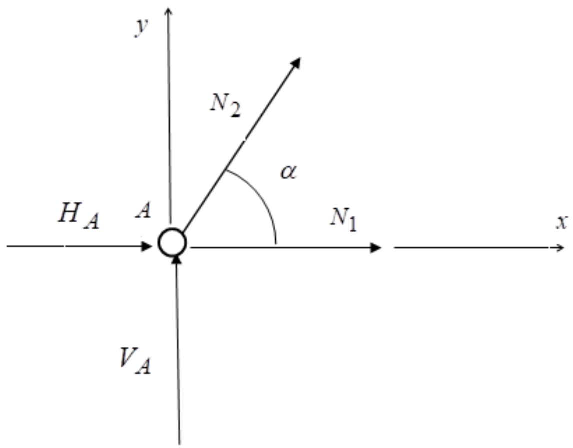


Fig. 2.9. The force scheme of joint A.

From the second of these equations, we determine N_2:

$$N_2 = -\frac{V_A}{\sin\alpha} = -\frac{V_A}{\sin 55^0},$$

or

$$N_2 = -\frac{49{,}64}{0{,}81915} = -60{,}6 \text{ kN.}$$

Since we obtain the answer with a "minus" sign, it means that in reality bar 2 is not stretched, but it is compressed, But we get the correct result that $N_2^* = 60{,}6$ kN, the bar 2 is compressed. Then from the first equation we can write:

$$N_1 = -H_A - N_2 \cos\alpha,$$

or

$$N_1 = -20 + 60 \cdot 0{,}573576 = 14{,}76 \text{ kN}.$$

Since here the answer is with a "plus" sign, then bar 1 is stretched.

Let us consider the equilibrium of the node C (Fig. 2.10). Since bar 2 is compressed, it presses on the hinge, as shown in Fig. 2.10. Write down the static equation of the node C:

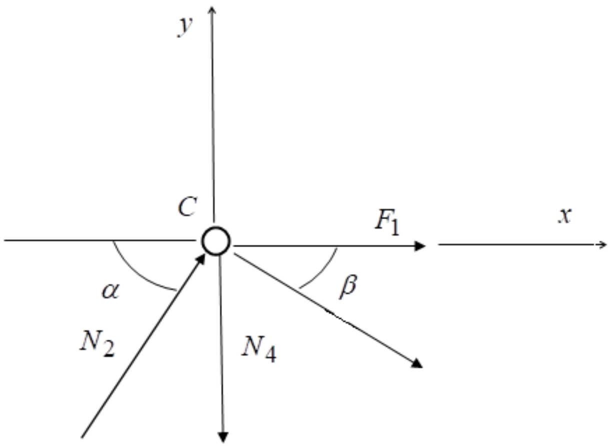


Fig. 2.10. The force scheme of joint C.

1. $\sum\limits_{i=1}^{n} F_{ix} = 0;$ $\quad \overset{*}{N_2} \cos\alpha + N_3 \cos\beta - F_1 = 0.$

2. $\sum\limits_{i=1}^{n} F_{iy} = 0;$ $\quad \overset{*}{N_2} \sin\alpha - N_3 \sin\beta - N_4 = 0.$

From the first equation we get:

$$N_3 = \frac{1}{\cos\beta}\left(F_1 - \overset{*}{N_2} \cos\alpha\right),$$

or it will be equal numerically:

$$N_3 = \frac{1}{0{,}90291}(20 - 60{,}6 \cdot 0{,}573576) = -16{,}3457 \text{ kN}.$$

Thus, the bar 3 is compressed by force $N_3^* = 16{,}3457$ kN.
Then from the second equation we get:

$$N_4 = N_2^* \sin\alpha - N_3 \sin\beta,$$

or it will be equal numerically:

$$N_4 = 60{,}6 \cdot 0{,}819152 + 16{,}3457 \cdot 0{,}42983 = 56{,}6665 \text{ kN}.$$

So, we get that $N_4 = 56{,}6665$ kN, the bar 4 is tensioned.

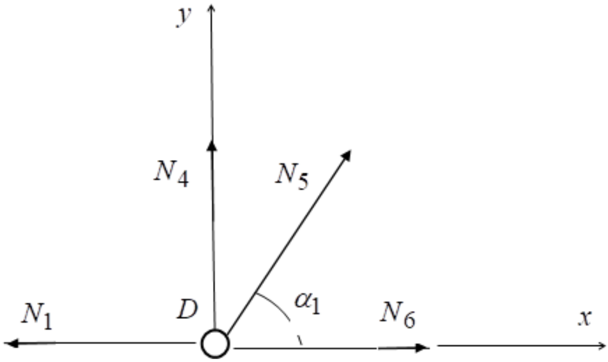


Fig. 2.11. The force scheme of joint D.

Let us go to considering the equilibrium of the node D (Fig. 2.11). We are going to find the unknowns N_5 and N_6. For this, we write the equilibrium equation:

1. $\sum_{i=1}^{n} F_{ix} = 0; \qquad N_5 \cos\alpha_1 + N_6 - N_1 = 0.$

2. $\sum_{i=1}^{n} F_{iy} = 0;\qquad N_5 \sin\alpha_1 + N_4 = 0.$

From the second equation, we get:

$$N_5 = -\frac{N_4}{\sin\alpha_1},\quad \text{or}\qquad N_5 = -\frac{56{,}6665}{0{,}689548} = -82{,}1792 \text{ kN.}$$

So, we get that $N_5^* = 82{,}1792$ kN, the bar 5 is compressed.
From the first equation, we get:

$$N_6 = N_1 - N_5 \cos\alpha_1 = 14{,}76 + 82{,}1792 \cdot 0{,}72424 = 74{,}277 \text{ kN.}$$

The bar 6 is tensioned, $N_6 = 74{,}277$ kN.

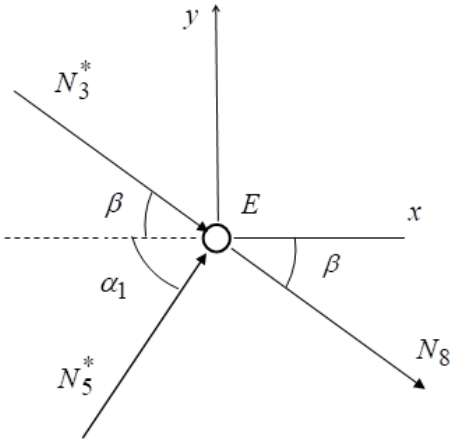


Fig. 2.12. The force scheme of joint E.

Let us consider the equilibrium of the node E (Fig. 2.12). In this case, the equations of static have the following form:

1. $\sum_{i=1}^{n} F_{ix} = 0;\qquad N_5^* \cos\alpha_1 + N_3^* \cos\beta + N_8 \cos\beta = 0.$

2. $\sum\limits_{i=1}^{n} F_{iy} = 0$; $N_5^* \sin\alpha_1 - N_7 - N_3^* \sin\beta - N_8 \sin\beta = 0$.

From the first equation, we determine the unknown force N_8:

$$N_8 = -\frac{1}{\cos\beta}\left(N_5^* \cos\alpha_1 + N_3^* \cos\beta\right) = -N_3^* - N_5^* \frac{\cos\alpha_1}{\cos\beta},$$

or:

$$N_8 = -16{,}3457 - 82{,}1792 \frac{0{,}724240}{0{,}902910} = -82{,}263 \text{ kN.}$$

Thus, we see, that the bar 8 is compressed, because we get $N_8^* = 82{,}263$ kN.

Than we are going to fine the effort N_7 from the second equation:

$$N_7 = N_3^* \sin\beta - N_8 \sin\beta - N_5^* \sin\alpha_1,$$

or:

$$N_7 = 82{,}1792 \cdot 0{,}689548 + 82{,}263 \cdot 0{,}42983 - 16{,}3457 \cdot 0{,}42983 = 85 \text{ kN.}$$

It is clear, that the bar 7 is tensioned.

We are going to consider the equilibrium of the node M to determine the forces in bars 9 and 10 (Fig. 2.13). To determine the unknowns forces N_9 and N_{10} we write down the equilibrium equation:

1. $\sum\limits_{i=1}^{n} F_{ix} = 0$; $N_9 \cos\alpha_2 - N_6 + N_{10} = 0$.

2. $\sum\limits_{i=1}^{n} F_{iy} = 0$; $N_7 + N_9 \sin\alpha_2 - F_2 = 0$.

From the second equation, we find effort N_9:

$$N_9 = \frac{1}{\sin\alpha_2}(F_2 - N_7),$$

or

$$N_9 = \frac{1}{0,42983}(50-85) = -81,4275 \text{ kN}.$$

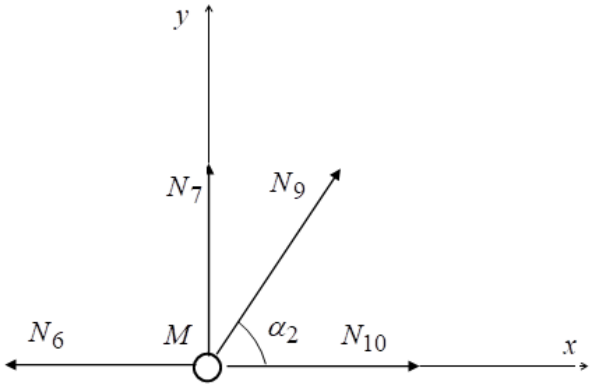


Fig. 2.13. The force scheme of joint M .

Thus, we see, that the bar 9 is compressed, because we get $N_9^* = 81,4275$ .kN

Than we find the force N_{10} from the first equation:

$$N_{10} = N_6 - N_9 \cos\alpha_2,$$

or

$$N_{10} = 74,277 + 81,4275 \cdot 0,90291 = 147,8 \text{ kN}.$$

It is clear, that the bar 10 is tensioned.

We determine the forces in bars 11 and 12, if we will consider the equilibrium of the node K (Fig. 2.14). In this case, we have:

1. $\sum\limits_{i=1}^{n} F_{ix} = 0;$ $\qquad N_9^* \cos\alpha_2 + N_8^* \cos\beta + N_{12} \cos\beta = 0.$

2. $\sum_{i=1}^{n} F_{iy} = 0$; $\quad N_9^* \sin\alpha_2 - N_8^* \sin\beta - N_{12} \sin\beta - N_{11} = 0$.

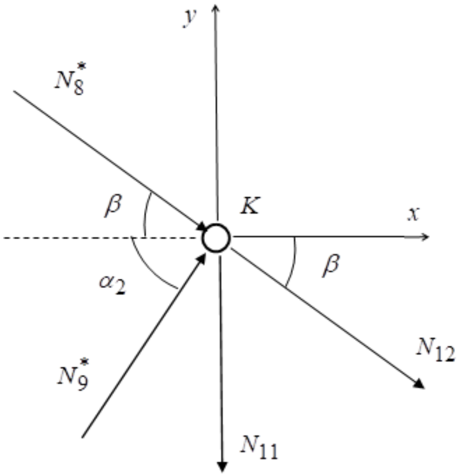


Fig. 2.14. The force scheme of joint K.

Since we can write that $\cos\alpha_2 = \cos\beta$, then the first equilibrium equation is reduced and it takes the form:

$$N_{12} = -N_9^* - N_8^*,$$

or

$$N_{12} = -82{,}263 - 81{,}4275 = -163{,}69 \text{ kN}.$$

Thus, we see, that the bar 12 is compressed, because we can write $N_{12}^* = 163{,}69$ kN.

From the second equation we defined the effort of bar 11, using that $\sin\alpha_2 = \sin\beta$. We get:

$$N_{11} = \left(N_9^* - N_8^* - N_{12}\right)\sin\beta,$$

or in numerical form we have:

$$N_{11} = (-82{,}263 + 81{,}4275 + 163{,}69)0{,}42983 = 70 \text{ kN}.$$

So, we obtained that, $N_{11} = 70$ kN. It is clear, that bar 11 is tensioned.

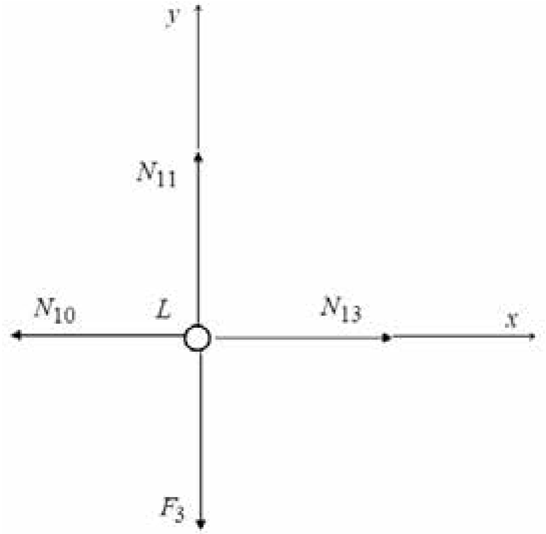


Fig. 2.15. The force scheme of joint L.

Let us consider the equilibrium of node L, where the effort in bar 13 is unknown (Fig. 2.15). Write down the equilibrium equation:

1. $\sum\limits_{i=1}^{n} F_{ix} = 0;\qquad -N_{10} + N_{13} = 0.$

2. $\sum\limits_{i=1}^{n} F_{iy} = 0;\qquad N_{11} - F_3 = 0.$

From the first, equation we get:

$$N_{13} = N_{10}, \text{ or } \qquad N_{13} = 147{,}8 \text{ kN}.$$

The bar 13 is tensioned.
The second equation we can rewrite such that:

$$70 - 70 = 0.$$

We check the determined efforts by considering the equilibrium of the node B (Fig. 2.16). We write down the equilibrium equation:

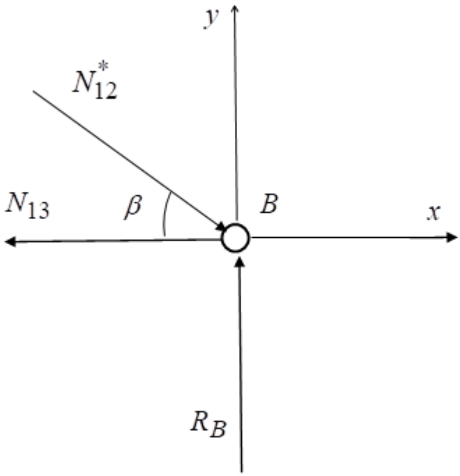


Fig. 2.16. The force scheme of joint B.

1. $\sum_{i=1}^{n} F_{ix} = 0;\qquad N_{12}^* \cos\beta - N_{13} = 0.$

2. $\sum_{i=1}^{n} F_{iy} = 0;\qquad R_B - N_{12}^* \sin\beta = 0.$

These equations include all previously defined parameters. Let us substitute their values.

1. $163{,}69 \cdot 0{,}90291 - 147{,}8 = 147{,}8 - 147{,}8 = 0.$

2. $70{,}36 - 163{,}69 \cdot 0{,}42983 = 70{,}36 - 70{,}359 \approx 0.$

Both equalities (equations) are fulfilled within rounding errors. This shows that the calculations are done correctly.

Self-control questions

1. *What kind of construction is truss?*
2. What is the concept of method of joints?
3. What is the base method of installation use in kinetic analysis for a plane truss?
4. Where are the loads of truss place?
5. How many unknown forces we can find from consideration one cutting joint?
6. What kind of force we have in truss bar?
7. What kind of deformation the truss bar has?

THEME 2.
THE METHOD OF SECTIONS

Sometimes the Ritter's method is called such as the method of sections.

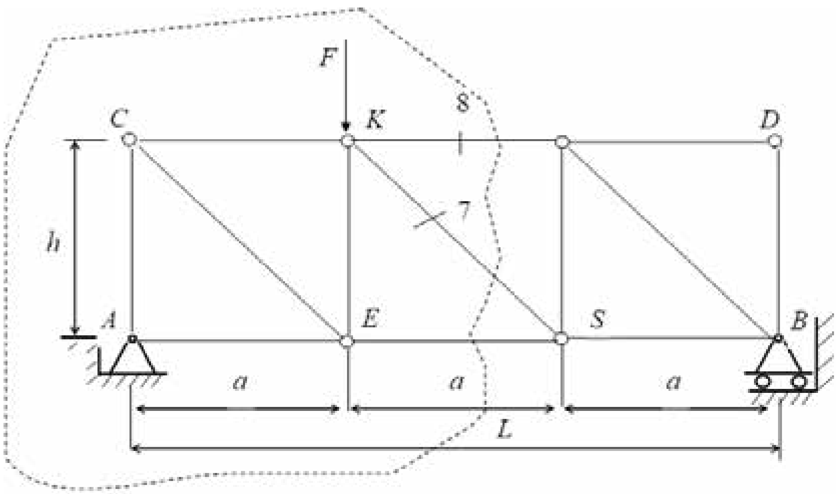


Fig. 2.17. The context of method of section.

According to this method, the equations of moments relative to moment points (so-called Ritter's points) or equations of projections on coordinate axes are formed for the selected fragment of truss. The moment point method is used when it is possible to divide the truss into two disks, while cutting three bars, the axes of which do not intersect at the same point (Fig. 2.17).

From the equilibrium conditions of one of the resulting discs, any of the three unknown forces in the cut bars can be determined. For this, it is necessary to use the sum of the moments of all the forces acting on the disc relative to the moment point (the intersection of the axes of the other two cut bars), as in Fig. 2.18. If the moment point is located at infinity, then all forces must be projected onto an axis perpendicular to these two rods — the y-axis to be determined in fig. 2.18.

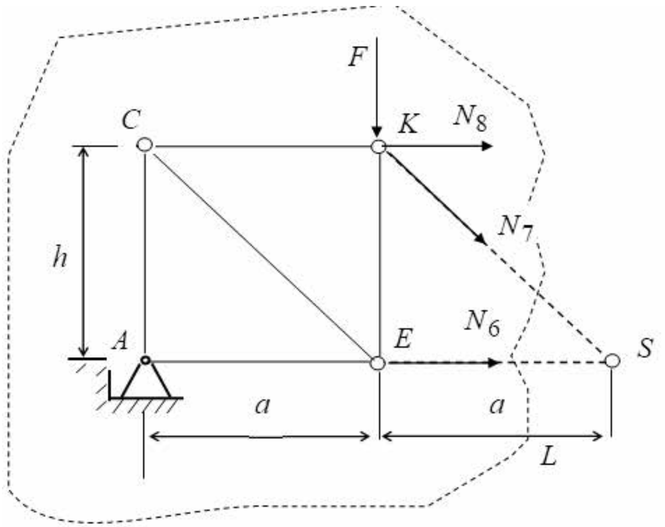


Fig. 2.18. The left part of truss shown on fig. 2.17.

Using the Ritter's method, let us consider an example of calculation a flat truss.

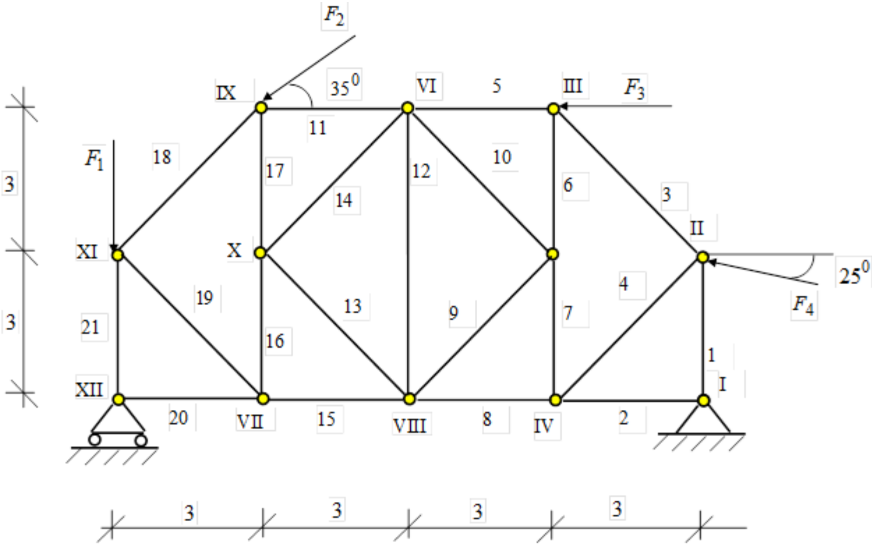


Fig. 2.19. The calculation scheme of truss.

Using the geometric dimensions of the given truss (Fig. 2.19), we have to determine the forces in bars 2, 3 and 4 by the Ritter's method. The following forces are applied to the truss: $F_1 = 20$ kN, $F_2 = 20$ kN, $F_3 = 30$ kN, $F_4 = 10$ kN.

Let us select the forces F_2 and F_4 into horizontal and vertical projections (Fig. 2.20), which are numerically equal:

$$F_{2г} = F_2 \cos 35^0 = 20 \cdot 0{,}8192 = 16{,}384 \text{ kN};$$

$$F_{2в} = F_2 \sin 35^0 = 20 \cdot 0{,}5736 = 11{,}472 \text{ kN};$$

$$F_{4г} = F_4 \cos 25^0 = 10 \cdot 0{,}9063 = 9{,}063 \text{ kN};$$

$$F_{4в} = F_4 \sin 25^0 = 10 \cdot 0{,}4226 = 4{,}226 \text{ kN}.$$

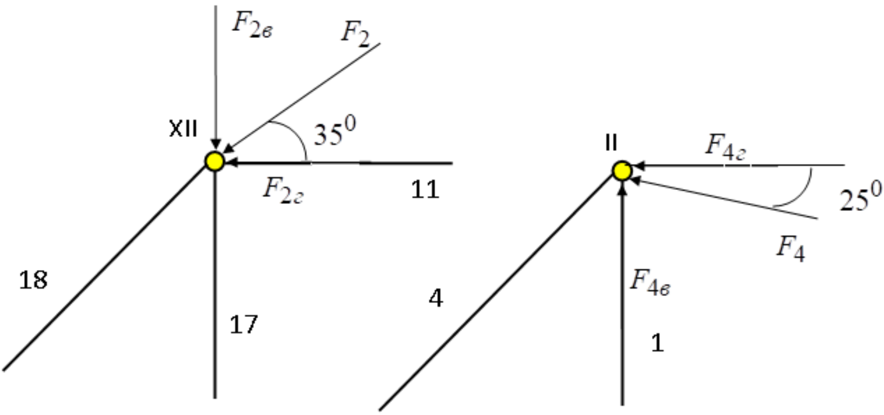


Fig. 2.20. The selection of forces F_2 and F_4 into horizontal and vertical projections.

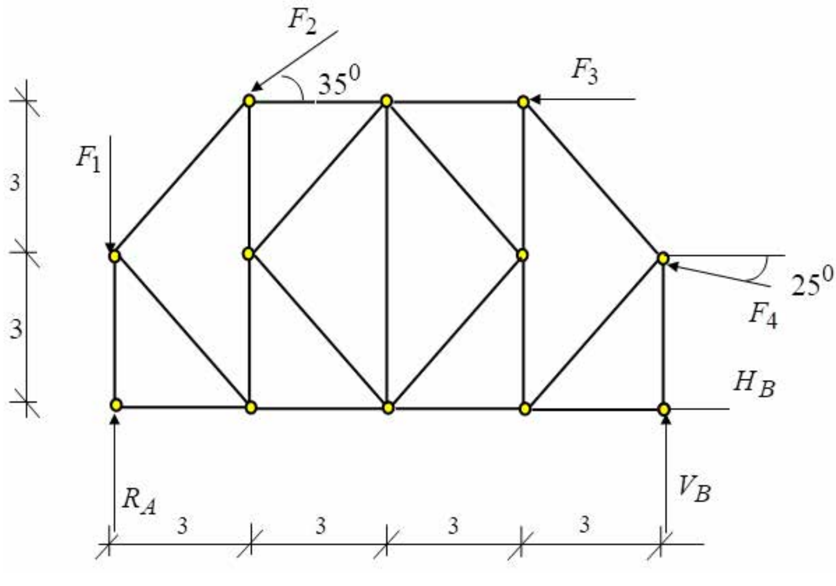


Fig. 2.21. The truss with reaction of supports.

We remove the supports of the truss and apply the appropriate reactions (Fig. 2.21).

We will determine the reactions of the supports from the following equilibrium equations:

1) $\sum F_x = 0$, $H_B - F_{4г} - F_{2г} - F_3 = 0$.

Where:

$$H_B = F_{4г} + F_{2г} + F_3 =$$

$$= 9{,}063 + 16{,}384 + 30 = 55{,}447 \text{ kN}.$$

2) $\sum M_B = 0$,

$$-R_A \cdot 12 + F_1 \cdot 12 + F_{2г} \cdot 6 + F_{2в} \cdot 9 + F_3 \cdot 6 + F_{4г} \cdot 3 = 0.$$

Where:

$$R_A = \frac{F_1 \cdot 12 + F_{2г} \cdot 6 + F_{2в} \cdot 9 + F_3 \cdot 6 + F_{4г} \cdot 3}{12} =$$

$$= \frac{20 \cdot 12 + 16{,}384 \cdot 6 + 11{,}472 \cdot 9 + 30 \cdot 6 + 9{,}063 \cdot 3}{12} = 54{,}062$$

kN.

3) $\sum M_A = 0$,

$$V_B \cdot 12 + F_{4г} \cdot 3 + F_{4в} \cdot 12 + F_3 \cdot 6 + F_{2г} \cdot 6 - F_{2в} \cdot 3 = 0.$$

Then:

$$V_B = \frac{-F_{4г} \cdot 3 - F_{4в} \cdot 12 - F_3 \cdot 6 - F_{2г} \cdot 6 + F_{2в} \cdot 3}{12} =$$

$$= \frac{-9{,}063 \cdot 3 - 4{,}226 \cdot 12 - 30 \cdot 6 - 16{,}384 \cdot 6 + 11{,}472 \cdot 3}{12} = -26{,}816$$

kN.

We run an analytical check using the following equilibrium equation:

$$\Sigma F_y = 0, \quad V_B + R_A + F_{4в} - F_{2в} - F_1 =$$

$$= -26{,}816 + 54{,}062 + 4{,}226 - 11{,}472 - 20 =$$

$$= 58{,}288 - 58{,}288 = 0.$$

Let us use Ritter's method to determine efforts N_2, N_3, N_4. To do this, we will use the cross-section of the truss along bars 2, 3 and 4, as shown in Fig. 2.22.

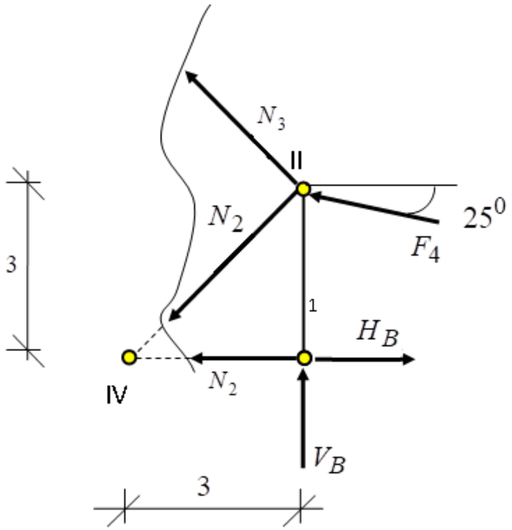


Fig. 2.22. The left part of truss shown on fig. 2.19.

If we choose joint II as the Ritter's point, then we can write:

$$\Sigma M_{II} = 0, \ H_B \cdot 3 - N_2 \cdot 3 = 0,$$

Where

$$N_2 = H_B = 55{,}447 \text{ kN}.$$

If we choose joint IV as the Ritter's point and we will determine the forces N_3:

$\sum M_{IV} = 0$,

$$V_B \cdot 3 + F_{4г} \cdot 3 + F_{4в} \cdot 3 + N_3 \cdot 3 \cdot \sin 45^0 + N_3 \cdot 3 \cdot \cos 45^0 = 0$$

.

Звідки:

$$N_3 = -\frac{V_B + F_{4г} + F_{4в}}{\sin 45^0 + \cos 45^0} =$$

$$= -\frac{-26{,}816 + 4{,}226 + 9{,}063}{2 \cdot 0{,}707} = 9{,}566 \text{ kN.}$$

If we project all the forces on the axis y, we will get:

$\sum F_y = 0$, $\quad V_B + F_{4в} - N_4 \cdot \sin 45^0 + N_3 \cdot \sin 45^0 = 0$,

Where:

$$N_4 = \frac{V_B + F_{4в} + N_3 \cdot \sin 45^0}{\sin 45^0} =$$

$$= \frac{-26{,}816 + 4{,}226 + 9{,}566 \cdot 0{,}707}{0{,}707} = -22{,}386 \text{ kN.}$$

Finally, we get that $N_2 = 55{,}447$ kN, $N_3 = 9{,}566$ kN, $N_4 = -22{,}386$ kN.

Self-control questions

1. *What is the context of the method of sections?*
2. *What is Ritter's point?*
3. *When is it advisable to use the method of section?*
4. *How many unknown forces can enter the section?.*

THEME 3.
THE METHOD OF COMPATIBLE SECTIONS

In some simple trusses, to determine the internal force in the bar such as N_1 (Fig. 2.23), it is necessary to use two cross-sections: Ritter's cross-section and cut the joint. Such an algorithm is called the method of compatible sections.

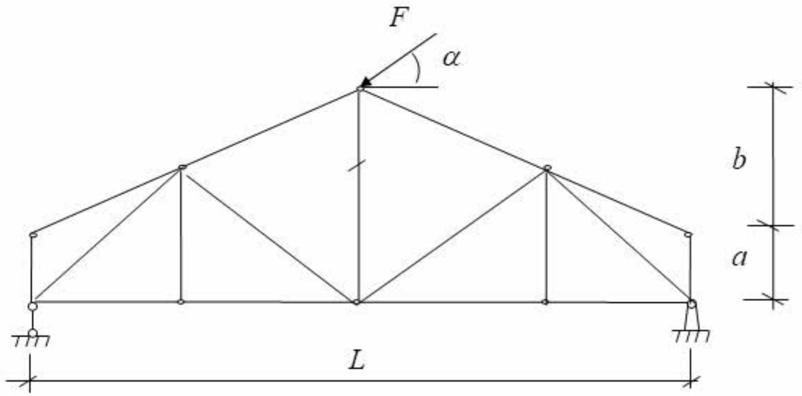


Fig. 2.23. The context of method of compatible sections.

Let us consider an example of calculation of a truss by the method of compatible cross-sections.

Let an external load be applied to the truss according to Fig. 2.24.

Firstly, we have to find reaction of supports:

$$\sum F_x = 0, \quad H_A + 12\cos 75^0 = 0, \; H_A = 3{,}11 \text{ kN.}$$

$$\sum M_A = 0, \; R_B + 12{,}6 - 10 \cdot 8{,}4 - 8 \cdot 4{,}2 -$$

$$- 12 \cdot 1{,}5\cos 75^0 - 12 \cdot 2{,}1\sin 75^0 = 0,$$

$$R_B = 11{,}63 \text{ kN}.$$

$$\sum M_B = 0,\ -V_A + 12{,}6 + 8 \cdot 8{,}4 + 10 \cdot 4{,}2 -$$

$$-12 \cdot 1{,}5 \cos 75^0 + 12 \cdot 10{,}5 \sin 75^0 = 0,$$

$$V_A = 17{,}96 \text{ kN}.$$

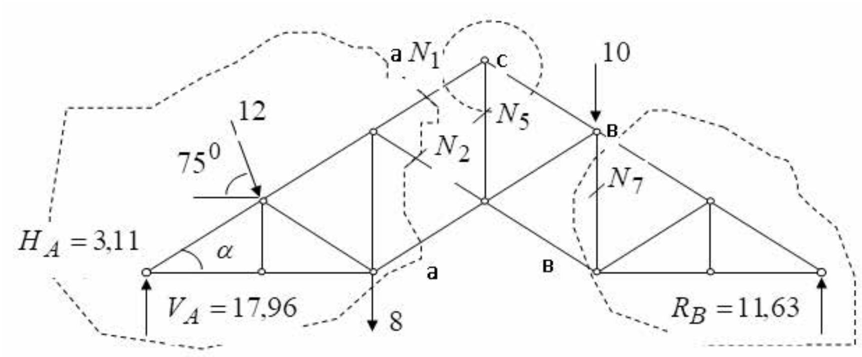


Fig. 2.24. The truss with reaction of supports.

Let us check the found reactions:

$$\sum F_y = 0, \quad R_B + V_A - 12 \sin 75^0 - 10 - 8 =$$

$$= 17{,}96 + 11{,}63 - 12 \cdot 0{,}9659 - 18 = 29{,}59 - 29{,}59 = 0.$$

We will find the efforts N_1 and N_2 from the static equations for section a-a (Fig. 2.25):

$$\sum M_K = 0,\ -1{,}5 N_1 \cdot \cos\alpha - 2{,}1 N_1 \sin\alpha + 17{,}96 \cdot 6{,}3 + 8 \cdot 2{,}1 +$$

$$+12 \cdot 4{,}2 - 3{,}11 \cdot 1{,}5 = 0,$$

Where

$$N_1 = -21{,}44 \text{ kN}.$$

$$\Sigma M_D = 0, \; -3N_1 \cdot \cos\alpha - 3N_2 \cos\alpha + 17{,}96 \cdot 4{,}2 +$$

$$+12 \cdot 1{,}5 \cdot 0{,}2588 - 12 \cdot 2{,}1 \cdot 0{,}9659 = 0,$$

Where

$$N_2 = -1{,}39 \text{ kN}.$$

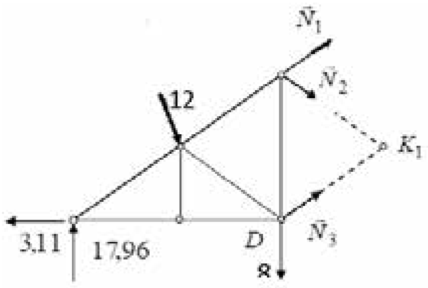


Fig. 2.25. The left part of truss shown on fig. 2.24.

If we consider the equilibrium of joint C, we can find the efforts N_5 (Fig. 2.26):

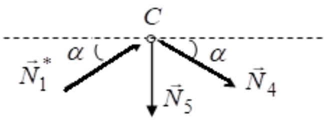


Fig. 2.26. The force scheme of joint C.

$N_1^* = -N_1 = 21{,}44$ kN.

$\Sigma F_x = 0, \quad N_1^* \cos\alpha + N_4 \cos\alpha = 0, \qquad N_4 = -N_1^* = -21{,}44$ kN.

$\Sigma F_y = 0, \quad N_1^* \sin\alpha - N_4 \sin\alpha - N_5 = 0, \quad N_5 = 24{,}9$ kN.

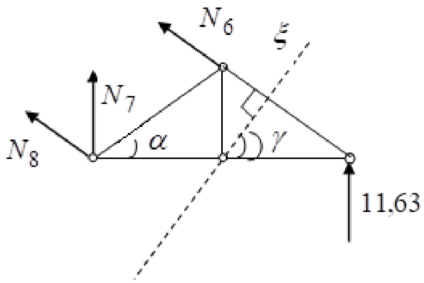


Fig. 2.27. The right part of truss shown on fig. 2.24.

We are going to find the effort N_7 from the static equation for a section в-в (Fig. 2.27).

$\sum\xi = 0$, $\qquad N_7 \cos\alpha + 11{,}63 \cdot \cos\alpha = 0$,

Where:

$$N_7 = -11{,}63 \text{ kN}.$$

Self-control questions

1. *What is the context of the method of compatible sections?*
2. *What are the cases is it advisable to use the method of compatible sections?*
3. *How many unknown forces can enter the section?*

THEME 4.
THE CALCULATION OF FLAT TRUSSES UNDER SNOW LOAD

In the previous paragraphs, we considered the methods of calculation of internal efforts in trusses under the so-called technological load. That is, it is a point load that is permanent and applied to the joints of the truss.

Snow load refers to the temporary load, which is usually given in the form of a uniformly distributed load per unit area (Fig. 2.28).

Fig. 2.28. An example of snow loads at truss roof.

The calculation of trusses under snow load runs according to SNiPu (standards, design norms) or BR (building regulations). In principle, the SNiP is mandatory, and the BPs have the character of recommendations, although the same thing is written in both documents. There is a difference between standard snow load and estimated snow load.

The standard snow load is the largest load that corresponds to normal operating conditions, which is calculated by deformations. The estimated snow load is the product of the standard load and the reliability factor. This coefficient takes into account the possible deviation of the standard snow load in the direction of increase in case of adverse circumstances and approximately it is is equal 1.4, that is, the calculated load is 40% more than the standard one.

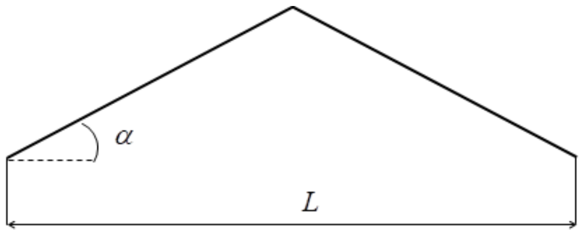


Fig. 2.29. The context of the coefficient c.

Thus, the estimated joint snow load, which is applied to the joints of the upper belt of the truss, is calculated according to the formula:

$$P_{CH} = 1{,}4 q_{CH} d \cdot b \cdot c, \quad (2.1)$$

where d is the length of the truss bars; b - the width of the truss bars; c - the coefficient of transition from the snow cover of the ground to the snow load on the cover, which is calculated according to SNiPu.

Intermediate values of the coefficient c is determined by linear interpolation. For a flat coating, from is equal to one. For two-slope coverage, the coefficient c depends on the slope (Fig. 2.29).

If the angle of slope is $\alpha \le 30^0$, then the coefficient $c = 1$. If we have that $30^0 < \alpha \le 45^0$, it means that $1 \ge c \ge 0{,}5$. In the case, when the angle of slope is $45^0 < \alpha \le 60^0$, then we get that $0{,}5 \ge c \ge 0$.

Let us consider one example of the calculation of the truss from the snow load. Let the bars of the upper belt of the truss shown in fig. 2.30, the applied snow load is $q_{CH} = 0{,}6$ kN/m^2, the width of the truss bars is equal to $h = 4{,}8$ m.

We have to calculate the forces in bars 9, 10, 11 and 15 in two cases: 1) the snow load is applied to the entire upper belt of the farm; 2) the snow load is applied to the left part of the truss.

According to Fig. 2.30 the length of the horizontal bars of the truss is:

$$d = \frac{12{,}6}{6} = 2{,}1 \text{ m.}$$

We will run a kinematic analysis of the given truss.

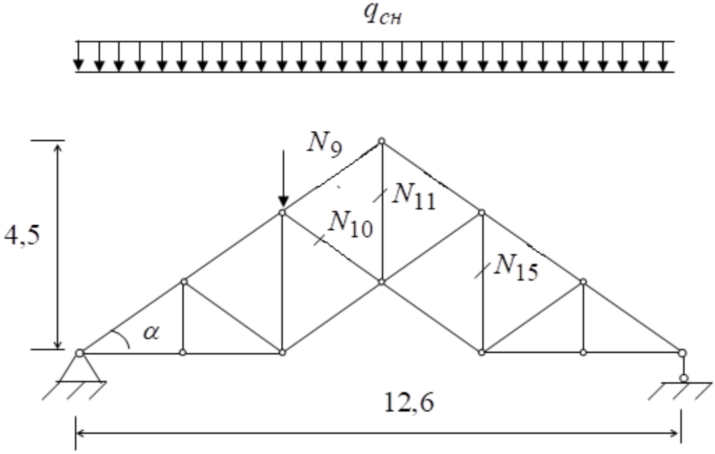


Fig. 2.30. The truss under the snow load.

1.1. Quantitative analysis.

Chebyshov's formula:

$$\Gamma = 2B - C. \qquad B = 12, \quad C = 24.$$

We can write down that:

$$\Gamma = 2 \cdot 12 - (5 + 11 + 5 + 3) = 24 - 24 = 0.$$

1.2. Qualitative analysis.

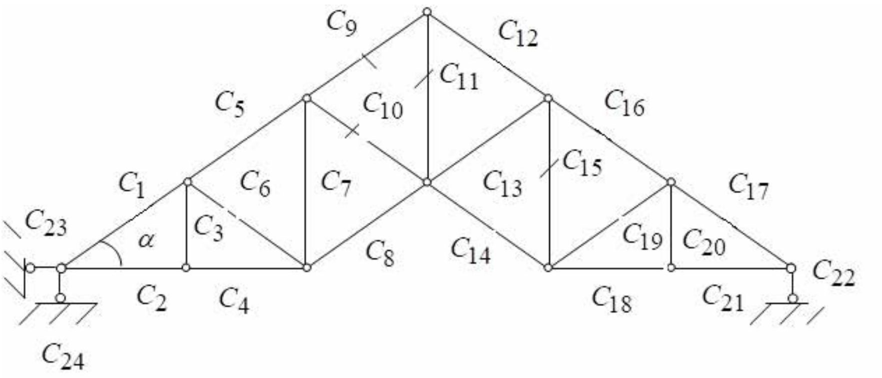


Fig. 2.31. The calculation scheme of truss shown on fig. 2.30.

From the first to the 11th stage of the assembly of calculation scheme is carried out using the "Dyad" method (Fig. 2.31). The 12th stage of installation is carried out according to the Polonzo's method (Fig. 2.32).

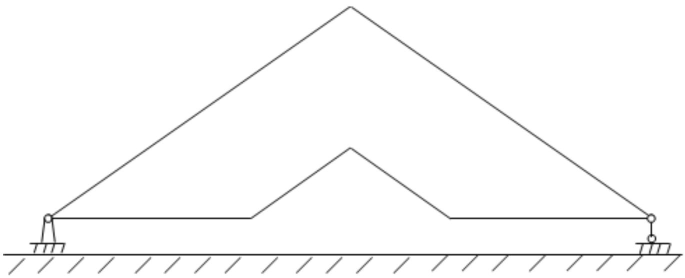


Fig. 2.32. The truss as a invariable disk.

2. Let us calculate the forces in bars 9th, 10th, 11th and 15th in the case, when the snow load is applied to the entire upper belt of the truss (Fig. 2.33).

   Then the joint load will be equal to:

$$P_{CH} = 1{,}4 q_{CH} d \cdot b \cdot c = 1{,}4 \cdot 0{,}6 \cdot 2{,}1 \cdot 4{,}8 \cdot 0{,}8 \approx 6{,}8 \text{ kN}.$$

Since the angle of slope is $\alpha = 35{,}5^0$, we get that the coefficient equals $c = 0{,}8$ respectively.

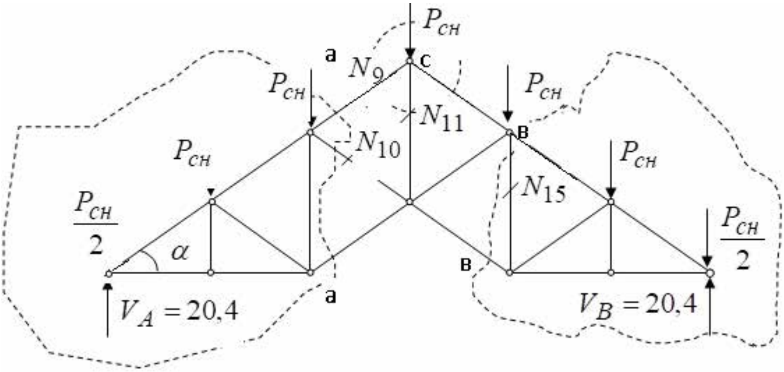


Fig. 2.33. The snow load is applied to the entire upper belt of the truss.

The reactions of the supports from the statics equations will be equal to:

$$V_A = 20{,}4 \text{ kN}, \quad V_B = 20{,}4 \text{ kN}.$$

We have to consider the equilibrium of the section a-a (Fig. 2.34):

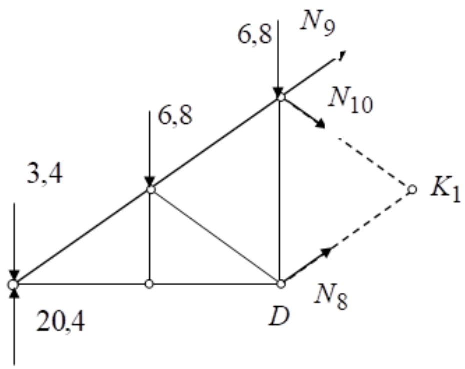


Fig. 2.34. The left part of given truss.

$$\sum M_K = 0,$$

$$-1{,}5 N_9 \cos\alpha - 2{,}1 N_9 \sin\alpha + 10{,}2 \cdot 6{,}3 - 20{,}4 \cdot 6{,}3 = 0,$$

$$N_9 = -26{,}33 \text{ kN}.$$

$$\sum M_D = 0,$$

$$-3 N_9 \cos\alpha - 3 N_{10} \cos\alpha + 3{,}4 \cdot 4{,}2 + 6{,}8 \cdot 2{,}1 - 20{,}4 \cdot 4{,}2 = 0,$$

$$N_{10} = 2{,}94 \text{ kN}.$$

Let us consider the equilibrium of the joint C (Fig. 2.35), taking into account that:

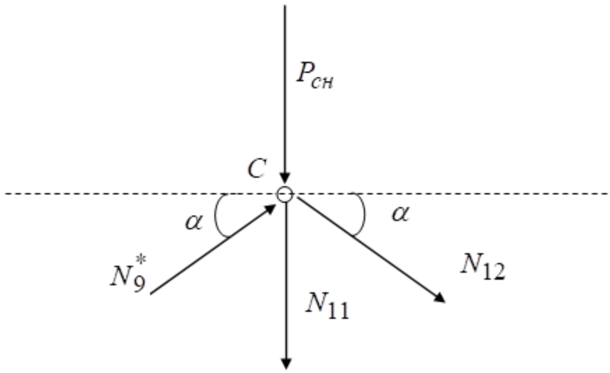


Fig. 2.35. The equilibrium of the joint C.

$$N_9^* = -N_9 = 26{,}33 \text{ kN}.$$

$$\sum F_x = 0, \quad N_9^* \cos\alpha + N_{12} \cos\alpha = 0, \quad N_{12} = -N_9^* = -26{,}33 \text{ kN}.$$

$$\sum F_y = 0, \quad N_9^* \sin\alpha - N_{12} \sin\alpha - N_{11} - P_{CH} = 0, \quad N_{11} = 23{,}78 \text{ kN}.$$

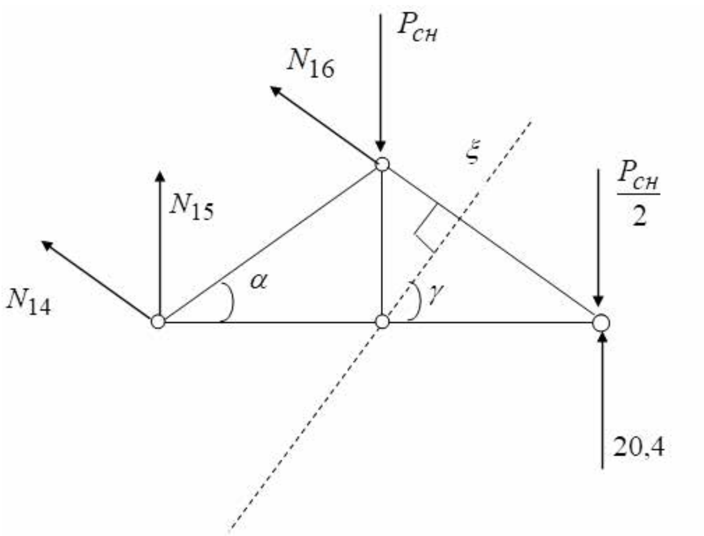


Fig. 2.36. The right part of given truss.

The force in 15th bar is determined from the equilibrium of the section в – в (Fig. 2.36), taken into account that:

$$\gamma = 90^0 - \alpha, \qquad \sin\gamma = \sin\alpha, \qquad \cos\gamma = \cos\alpha.$$

Finally we get that:

$$\Sigma\xi = 0, \qquad N_{15}\cos\alpha - 6{,}8\cos\alpha - 3{,}4\cos\alpha + 20{,}4\cos\alpha = 0,$$

$$N_{15} = -10{,}2 \text{ kN}.$$

It will be useful to consider the case, when the snow load is applied to the left part of the truss (Fig. 2.37).

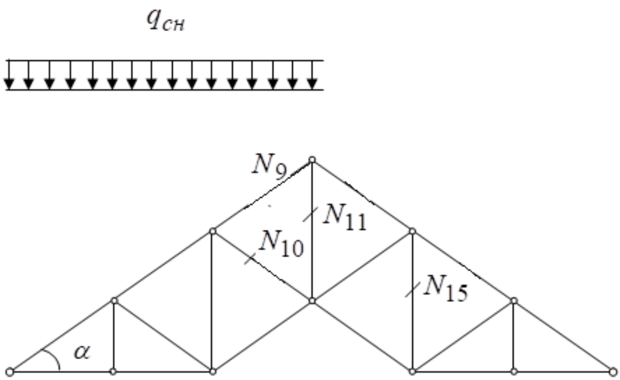


Fig. 2.37. The snow load is applied to the left part of the truss.

As in the previous example, the joint load will be equal to $P_{CH} \approx 6{,}8$ kN, but it will be applied only to the nodes of the upper belt of the left part of the truss, as shown in Fig. 2.38.

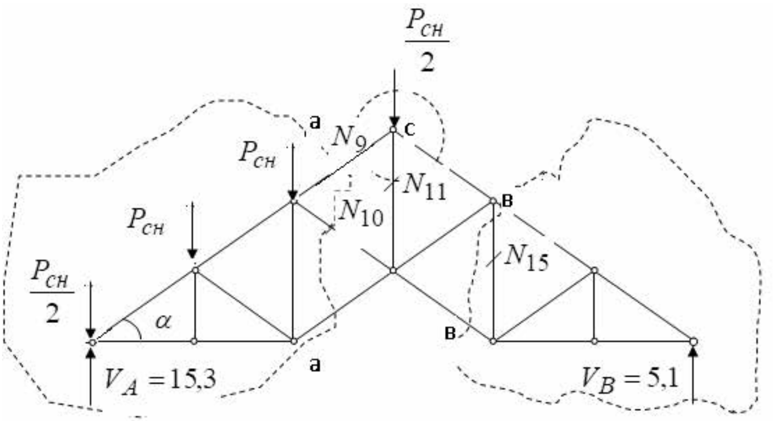


Fig. 2.38. The joint load is applied to only to the nodes of the upper belt of the left part of the truss.

In this case, the reactions of the supports will be equal:

$\Sigma M_A = 0,\ V_B \cdot 12{,}6 - 10{,}2 \cdot 6{,}3 = 0, \qquad V_B = 5{,}1$ kN.

$\Sigma F_y = 0, \quad V_B - 3P_{CH} + V_A = 0, \qquad V_A = 15{,}3$ kN.

Firstly, we have to consider the equilibrium of the cross-section a-a (Fig. 2.39):

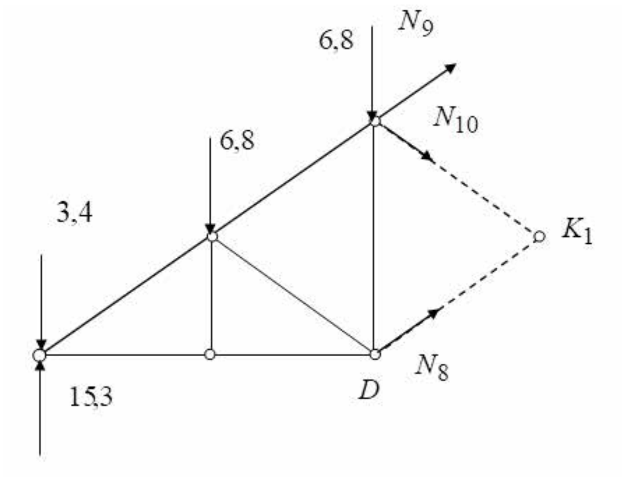


Fig. 2.39. The equilibrium of the cross-section a-a.

$$\Sigma M_K = 0, \qquad -1{,}5N_9 \cos\alpha - 2{,}1N_9 \sin\alpha + 10{,}2 \cdot 6{,}3 - 15{,}3 \cdot 6{,}3 = 0,$$

$$N_9 = -13{,}16 \text{ kN}.$$

$$\Sigma M_D = 0,$$
$$-3N_9 \cos\alpha - 3N_{10} \cos\alpha + 3{,}4 \cdot 4{,}2 + 6{,}8 \cdot 2{,}1 - 15{,}3 \cdot 4{,}2 = 0,$$

$$N_{10} = -1{,}46 \text{ kN}.$$

We determine the forces N_{11} from the equilibrium of the joint C (Fig. 2.40). As in the previous case, we taken into account that:

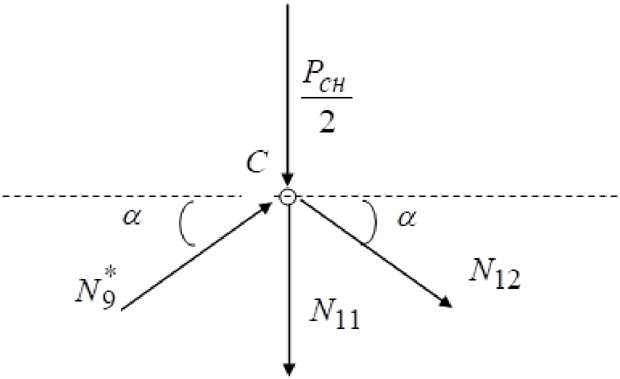


Fig. 2.40. The equilibrium of the joint C.

$$N_9^* = -N_9 = 13{,}16 \text{ kN}.$$

Then:

$$\Sigma F_x = 0, \quad N_9^* \cos\alpha + N_{12} \cos\alpha = 0, \quad N_{12} = -N_9^* = -13{,}16 \text{ kN}.$$

$$\Sigma F_y = 0, \quad N_9^* \sin\alpha - N_{12} \sin\alpha - N_{11} - \frac{P_{CH}}{2} = 0, \quad N_{11} = 11{,}88 \text{ kN}.$$

The force in 15th bar is determined from the equilibrium of the cross-section в – в (Fig. 2.41)

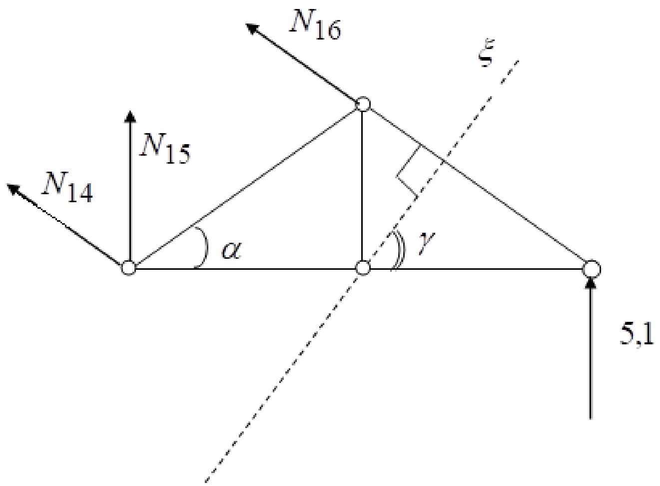


Fig. 2.41. The equilibrium of the cross-section в – в.

$$\Sigma\xi = 0, \quad N_{15}\cos\alpha + 5{,}1\cos\alpha = 0, \quad N_{15} = -5{,}1 \text{ kN}.$$

THEME 5.
THE CALCULATION OF FLAT TRUSSES ON RIGITY

We will remind that the stiffness calculation of a mechanical system means the calculation of deformations (displacements and rotation angles) of its components.

As you know, from the course of strength of materials, the main method of calculating the rigidity of bar structures, which include trusses, frames (flat and spatial), etc., is Mohr's integral, which in the general case has the form:

$$\delta_{iF} = \Sigma \int_{\ell} \frac{N_i N_F}{EA} dx + \Sigma \int_{\ell} \frac{M_i M_F}{EI} dx + \Sigma \int_{\ell} \frac{\nu Q_i Q_F}{GA} dx,$$

(2.2)

where δ_{iF} is the generalized displacement;

N_i, M_i, Q_i - corresponding internal forces in the bars of the system from the unit state;

N_F, M_F, Q_F - corresponding internal forces in the rods of the system due to the load condition;

EA, EI, GA - stiffness of the bars, respectively, in tension - compression, bending and shear.

As we have discussed earlier, the bars of flat trusses work mainly on tension-compression deformation. Therefore, in the case of calculating displacements in truss bars, the formula (2.2) has been reduced to the following form:

$$\delta_{iF} = \Sigma \int_{\ell} \frac{N_i N_F}{EA} dx. \quad (2.3)$$

It should be noted that the calculation of displacements in truss bars according to formula (2.3) is not complete, because it does not take into account temperature effects.

Self-control questions

1. *What is the type of load snow load to belong to?*
2. *Please, explain the meaning of the concept of joint snow load.*
3. *How does the joint snow load to apply to truss joints correctly?*
4. *Please, give an interpretation of the formula for joint snow load.*
5. *What is context of truss stiffness?*
6. *What is formula of the definition of truss bars deformation?*
7. *Does the formula (2.3) take into account temperature effects?*
8. *What is dimension of snow load?*
9. *What is the coefficient of transition?*

THE EXAMPLES OF INDIVIDUAL WORK
«THE CALCULATION OF FLAT TRUSS ON STRENGTH »

The students can get task of this work from table 2 in appendence.

Problem 2.1.

For a given flat truss (Fig. 2.42), it necessary run the following actions:

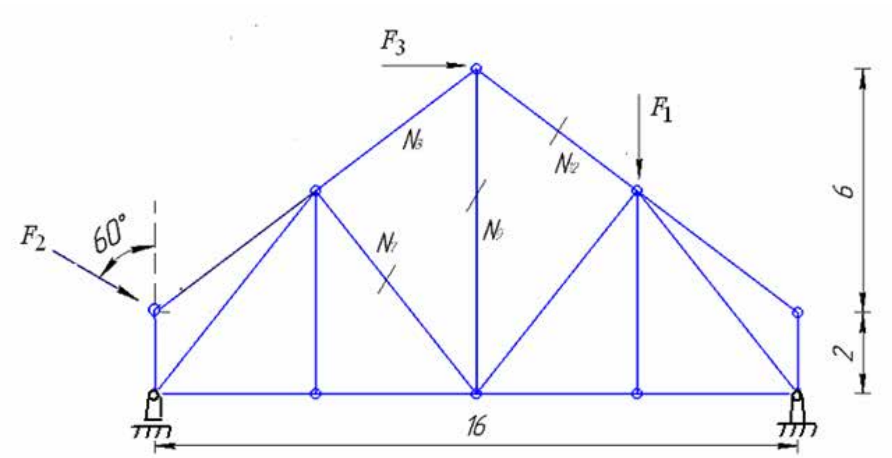


Fig. 2.42. The given truss in Problem 2.1.

1. Carry out a kinematic analysis of the given truss.
2. Determine the forces in the specified truss bars from the technological load, if $F_1 = 10$ kN, $F_2 = 20$ kN, $F_3 = 30$ kN (it run by the method of joints).
3. Check the obtained result using the method of sections or the cross-section compatibility method.
4. Determine the forces in the given bars under the snow load.
5. In the specified bars, determine the deformation due to the technical load.

Solution:

1. Kinematic analysis. The given calculation scheme contains 18 bars, 10 joints, a hinge and one disc. (2.43)

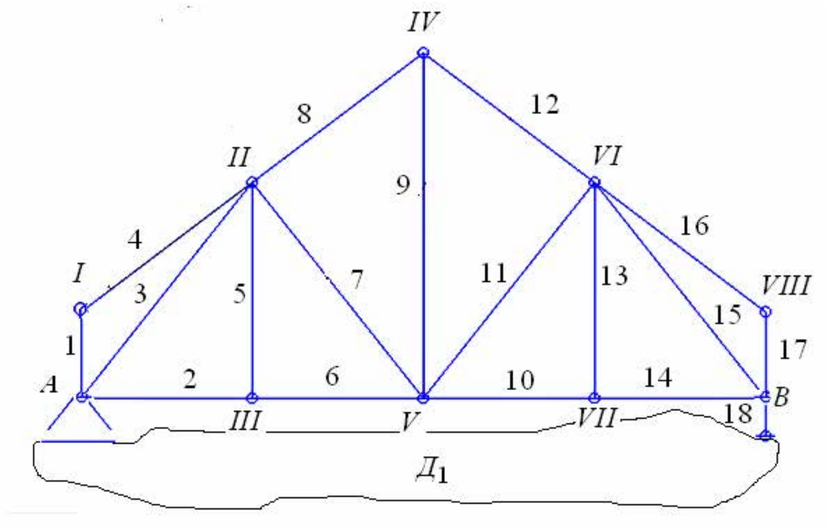


Fig. 2.43. The calculation scheme of truss in Problem 2.1.

1.1. Quantitative analysis.

$Д = 1, \quad В = 10, \quad Ш = 1, \quad С = 18.$

Then, according to the Chebyshov's formula, we get:

$$Г = 3 \cdot 1 + 10 \cdot 2 - 2 \cdot 1 - 18 - 3 = 23 - 23 = 0$$

Conclusion: the given construction is statically defined

1.2. Qualitative analysis.

The given calculation scheme is built in 10 stages, of which 1 - 9 are the Dyad method; 10 – Polonzo's method.

Thus, the given construction is geometrically invariable system.

2. Determination of resistance reactions (Fig. 2.44):

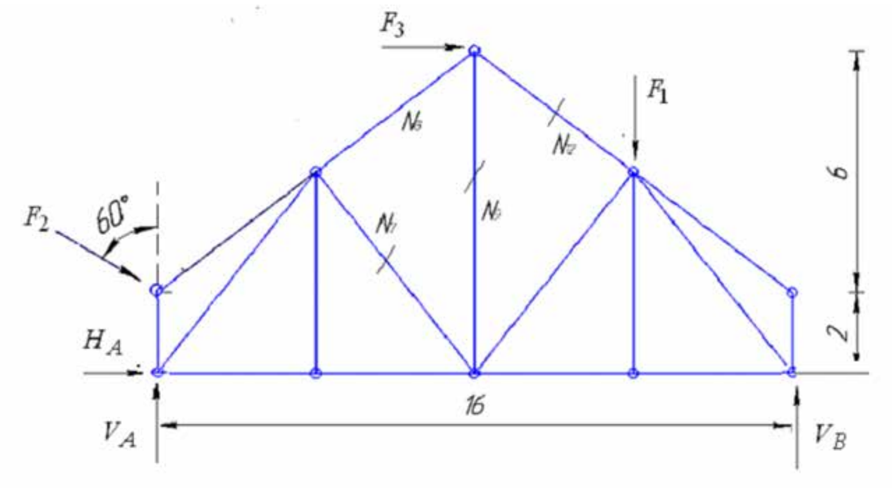


Fig. 2.44. The truss in Problem 2.1 with reactions.

1. $\sum F_{x_i} = 0, \qquad H_A + 20 \cdot \sin 60^0 + 30 = 0,$

$H_A = -20 \cdot \sin 60^0 - 30, \qquad H_A = -20 \cdot 0{,}866 - 30 = -47{,}3$ kN.

2. $\sum M_{A_i} = 0, \qquad V_B \cdot 16 - 10 \cdot 12 - 20 \cdot 2 \cdot \sin 60^0 - 30 \cdot 8 = 0$

$$V_B = \frac{10 \cdot 12 + 20 \cdot 2 \cdot \sin 60^0 + 30 \cdot 8}{16} = \frac{120 + 34{,}64 + 240}{16} = 24{,}66 \text{ kN.}$$

3. $\sum M_{B_i} = 0, \ -V_A \cdot 16 + 10 \cdot 4 - 20 \cdot 2 \cdot \sin 60^0 + 20 \cdot 16 \cdot \cos 60^0 -$
$- 30 \cdot 8 = 0,$

$$V_A = \frac{10 \cdot 4 - 240 - 40 \cdot \sin 60^0 + 320 \cdot \cos 60^0}{16} =$$

$$= \frac{40 - 240 - 34{,}64 + 160}{16} = -4{,}66 \text{ kN}.$$

Let us check the reactions, which had been found:

$$\Sigma F_{y_i} = 0, \quad V_A + V_B - 20 \cdot \cos 60^0 - 10 =,$$
$$= -4{,}66 + 24{,}66 - 20 \cdot 0{,}5 - 10 = 24{,}66 - 24{,}66 = 0.$$

So, the reactions are found correctly.

3. We will determine the desired forces in the bar of given truss by the method of joints.

3.1. Firstly, we have to consider the equilibrium of joint *I* (Fig. 2.45).

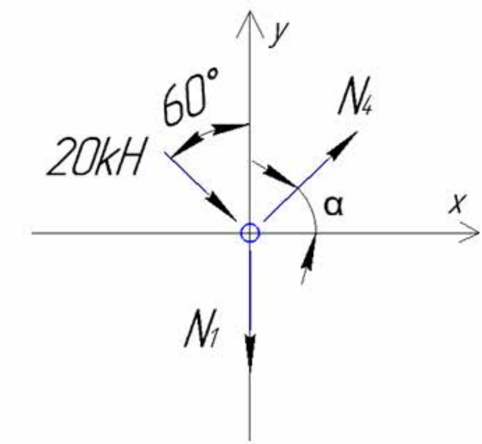


Fig. 2.45. The equilibrium of joint *I*.

From the geometry of the truss, we determine the characteristics of the angle α:

$$\sin \alpha = 0{,}6, \qquad \cos \alpha = 0{,}8.$$

Let us write down the equilibrium equation for the joint I:

$$\sum F_{x_i}^{I} = 0, \quad N_4 \cdot \cos\alpha + 20 \cdot \sin 60^0 = 0,$$

$$N_4 = -21{,}65 \text{ kN;}$$

$$\sum F_{y_i}^{I} = 0, \quad N_4 \cdot \sin\alpha - 20 \cdot \cos 60^0 + N_1 = 0,$$

$$N_1 = 20 \cdot \cos 60^0 - N_4 \cdot \sin\alpha,$$

$$N_1 = 20 \cdot 0{,}5 - 21{,}65 \cdot 0{,}6 = -22{,}99 \text{ kN.}$$

3.2. Consider the equilibrium of the joint A (Fig. 2.46).

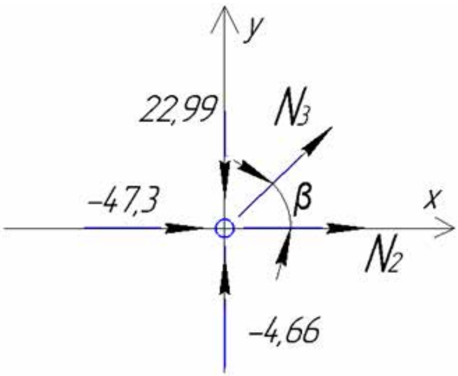


Fig. 2.46. The equilibrium of joint A.

From the geometry of the truss, we determine the characteristics of the angle β:

$$\sin\beta = 0{,}781, \quad \cos\beta = 0{,}625.$$

Then from static equation we get:

$$\sum F_{y_i}^{A} = 0, \quad -4{,}66 - 22{,}99 + N_3 \cdot \sin\beta = 0,$$

$$N_3 = 35{,}408 \text{ kN};$$

$$\sum F_{x_i}^{A} = 0, \quad -47{,}3 + N_3 \cdot \cos\beta + N_2 = 0,$$

$$N_2 = 20 \cdot 0{,}5 - 21{,}65 \cdot 0{,}6 = -22{,}99 \text{ kN}.$$

3.3. We will consider the equilibrium of the joint III (Fig. 2.47)

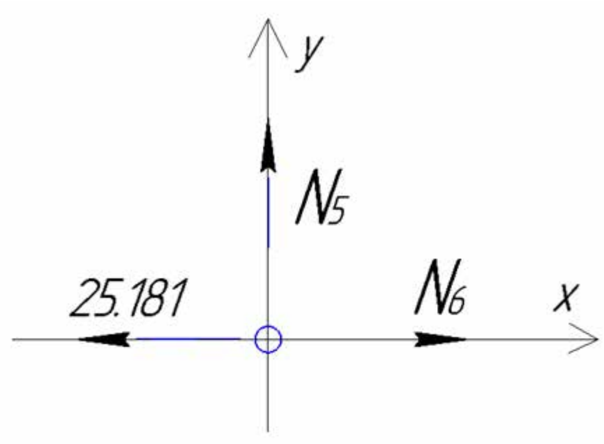


Fig. 2.47. The equilibrium of joint III.

$$\sum F_{x_i}^{III} = 0, \; -25{,}181 + N_6 = 0,$$

$$N_6 = 25{,}181 \text{ kN};$$

$$\sum F_{y_i}^{III} = 0, \; N_5 \equiv 0.$$

3.4. Consider the equilibrium of the joint II (Fig. 2.48).

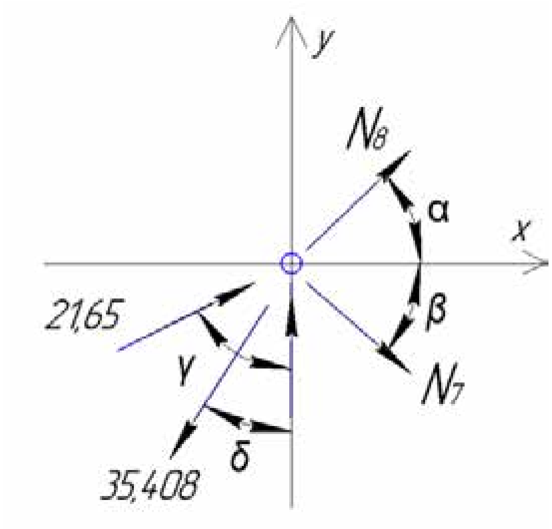


Fig. 2.48. The equilibrium of joint II.

$$\Sigma F_{x_i}^{II} = 0,\ 21{,}65 \cdot 0{,}8 - 35{,}408 \cdot 0{,}625 + N_7 \cdot 0{,}625 + N_8 \cdot 0{,}8 = 0, \quad (2.4)$$

$$\Sigma F_{y_i}^{III} = 0,\ 21{,}65 \cdot 0{,}6 - 35{,}408 \cdot 0{,}781 - N_7 \cdot 0{,}781 + N_8 \cdot 0{,}6 = 0, \quad (2.5)$$

If we solved the system of equation (3.4) and (3.5), we obtaine the values of efforts in bars 7 and 8:

$$N_7 = -8{,}853 \text{ kN}, \qquad N_8 = 12{,}912 \text{ kN}. \qquad (2.6)$$

3.5. Let us consider the equilibrium of the joint IV (Fig. 2.49).

From the geometry of the truss, we determine the characteristics of the angle γ:

$$\sin\gamma = 0{,}8, \qquad \cos\gamma = 0{,}6.$$

Then from static equation, we get:

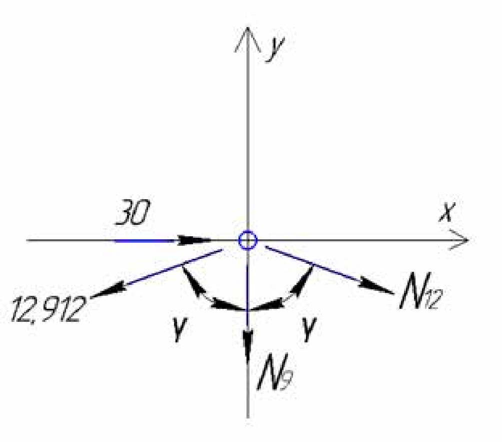


Fig. 2.49. The equilibrium of joint IV.

$$\sum F_{x_i}^{IV} = 0,\ 30 - 12{,}912 \cdot \sin\gamma + N_{12} \cdot \sin\gamma = 0,$$

$$N_{12} = -24{,}588 \text{ kN}; \qquad (2.7)$$

$$\sum F_{y_i}^{IV} = 0,\ -12{,}912 \cdot \cos\gamma - N_{12} \cdot \cos\gamma - N_9 = 0,$$

$$N_9 = -7{,}005 \text{ kN}. \qquad (2.8)$$

4. We are going to check the found efforts in the bars 7, 9 and 12 by the cross-section compatibility method.

4.1. Let us consider Ritter's section (Fig. 2.50):

$$\sum M_{V_i} = 0,\ - N_8 \cdot 5 \cdot \cos\alpha - N_8 \cdot 4 \cdot \sin\alpha + 4{,}66 \cdot 8 - 20 \cdot 2 \cdot \sin 60^0 + $$
$$+ 20 \cdot 8 \cdot \cos 60^0 = 0,$$

or

$$-N_8 \cdot 5 \cdot 0{,}8 - N_8 \cdot 4 \cdot 0{,}6 + 37{,}28 + 80 - 34{,}64 = 0,$$

where:

$$-N_8 \cdot 6{,}4 + 82{,}64 = 0, \text{ або } \quad N_8 = 12{,}91 \text{ kN}. \qquad (2.9)$$

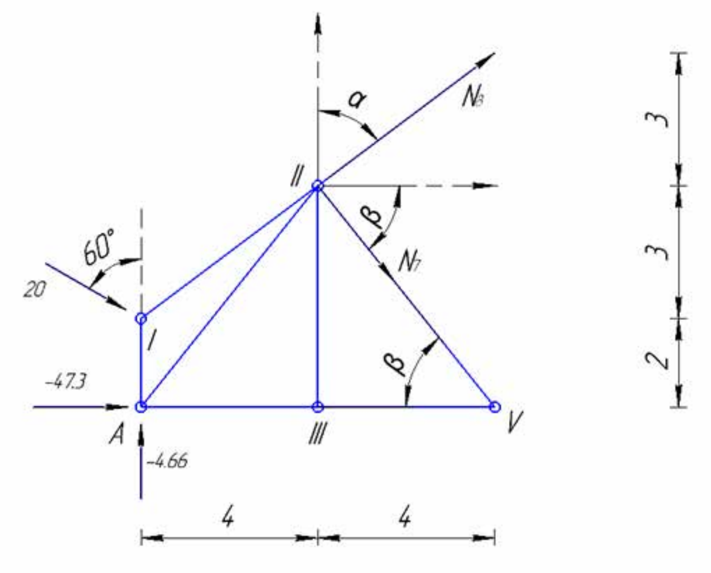


Fig. 2.50. Ritter's section of the truss in Problem 2.1.

$$\sum F_{y_i} = 0, \; N_8 \cdot \sin\alpha - 4{,}66 - 20 \cdot \cos 60^0 - N_7 \cdot \sin\beta = 0,$$

or

$$N_7 = \frac{N_8 \cdot \sin\alpha - 4{,}66 - 20 \cdot \cos 60^0}{\sin\beta},$$

where:

$$N_7 = \frac{12{,}912 \cdot 0{,}6 - 4{,}66 - 20 \cdot 0{,}5}{0{,}781} = -8{,}852 \text{ kN}. \qquad (2.10)$$

4.2. Let us consider the equilibrium of the joint IV (Fig. 2.51).

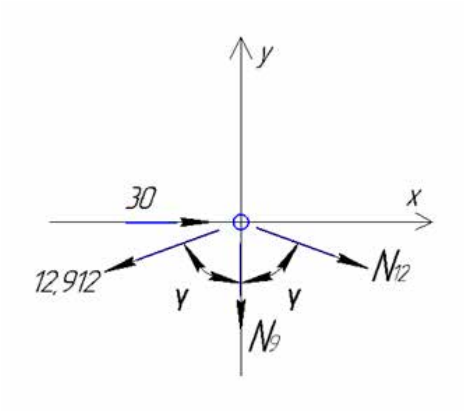


Fig. 2.51. The equilibrium of joint IV.

$$\sum F_{x_i}^{IV} = 0,\; 30 - 12{,}912 \cdot \sin\gamma + N_{12} \cdot \sin\gamma = 0,$$

$$N_{12} = -24{,}588 \text{ kN}; \qquad (2.11)$$

$$\sum F_{y_i}^{IV} = 0,\; -12{,}912 \cdot \cos\gamma - N_{12} \cdot \cos\gamma - N_9 = 0,$$

$$N_9 = -7{,}005 \text{ kN}. \qquad (2.12)$$

If we compare expressions (2.6) – (2.8) with expressions (2.10) – (2.11), it can be seen that the corresponding values of the required forces in the truss bars are equal to the second sign after the decimal point.

4. Determination of forces in truss bars from snow load.

Let a snow load with an intensity of $q = 0{,}5$ kN/m^2 is applied to the left part of the upper belt of the given truss (Fig. 2.52).

Determine the forces in bars 7th, 9th and 12th of the given truss under the snow load.

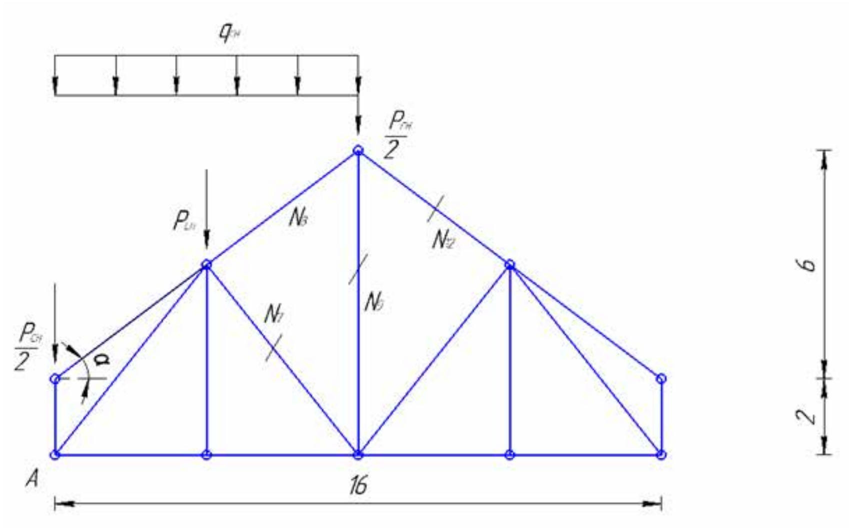


Fig. 2.52. The given truss under the snow load.

5.1. Given that $\alpha = 36{,}87^0$, we will find the coefficient c for calculation of the joint snow load according to formula (2.1). Since $30^0 \le \alpha \le 45^0$, the value of the coefficient c will be in the range from 1 to 5. To calculate it, we will use the interpolation method:

$$c = 1 - \frac{0{,}5}{15} \cdot 6{,}87 = 0{,}771. \qquad (2.13)$$

Then according to formula (2.1) we get:

$$P_{CH} = 1{,}4 \cdot 0{,}5 \cdot 0{,}771 \cdot 4 \cdot 0{,}24 = 0{,}518 \text{ kN}.$$

5.2. We attach the load P_{CH} to the joints of the upper belt of the left side of the truss according to fig. 2.53 and determine the reference reactions:

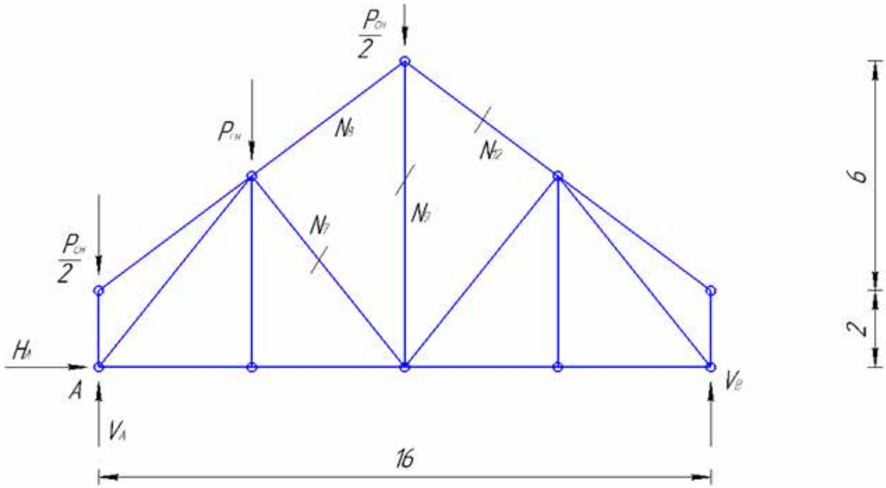


Fig. 2.53. The truss under the snow load with reactions.

1. $\Sigma F_{x_i} = 0, \qquad H_A \equiv 0,$

2. $\Sigma M_{A_i} = 0, \qquad V_B \cdot 16 - P_{CH} \cdot 4 - P_{CH} \cdot 4 = 0$

$$V_B = \frac{P_{CH} \cdot 8}{16} = \frac{P_{CH}}{2} = 0{,}259 \text{ kN.}$$

3. $\Sigma M_{B_i} = 0, \; -V_A \cdot 16 + P_{CH} \cdot 8 + P_{CH} \cdot 12 + P_{CH} \cdot 4 = 0$

$$V_A = \frac{P_{CH} \cdot 24}{16} = \frac{3}{2} P_{CH} = 0{,}777 \text{ kN.}$$

We will check the values of obtaine reactions:

$$\Sigma F_{y_i} = 0, \quad V_A + V_B - P_{CH} \cdot 2 = \frac{3P_{CH}}{2} + \frac{P_{CH}}{2} - P_{CH} \cdot 2 =$$

$$= 2P_{CH} - 2P_{CH} = 0.$$

So, the reactions are found correctly.

Using the cross-section compatibility method, we determine the forces in the marked truss rods from the snow load.

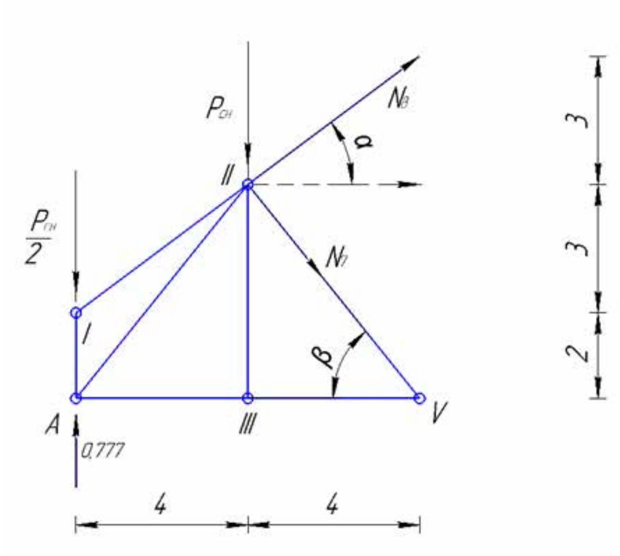


Fig. 2.54. The Ritter's section foe truss under the snow load.

Let us run the Ritter's section (Fig. 2.54)

$$\sum M_{V_i} = 0, \quad -N_8 \cdot 5 \cdot 0{,}8 - N_8 \cdot 4 \cdot 0{,}6 - 6{,}217 + 2{,}072 + 2{,}072 = 0,$$

where:

$$-N_8 \cdot 6{,}4 - 2{,}073 = 0, \text{ або } \quad N_8 = -0{,}324 \text{ kN}.$$

$$\sum F_{y_i} = 0,\ N_8 \cdot \sin\alpha - \frac{3}{2}P_{CH} + \frac{3}{2}P_{CH} - N_7 \sin\beta = 0,$$

or

$$N_7 = \frac{N_8 \cdot \sin\alpha}{\sin\beta},$$

where:

$$N_7 = \frac{-0{,}324 \cdot 0{,}6}{0{,}781} = -0{,}249 \text{ kN}.$$

4.2. We will consider the equilibrium of joint IV (Fig. 2.55).

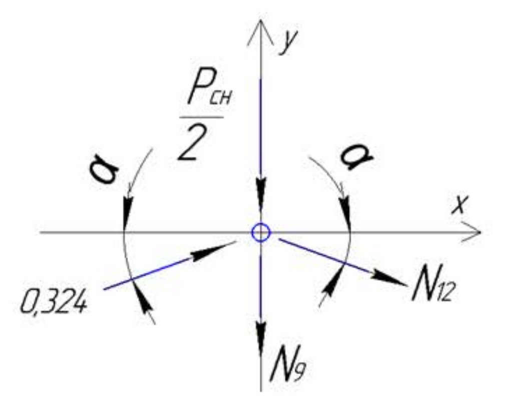


Fig. 2.55. The equilibrium of joint IV under snow load.

$$\sum F_{x_i}^{IV} = 0,\ 0{,}324 \cdot \cos\alpha + N_{12} \cdot \cos\alpha = 0,$$

$$N_{12} = -0{,}324 \text{ kN};$$

$$\sum F_{y_i}^{IV} = 0,\ 0{,}324 \cdot \sin\alpha + N_{12} \cdot \sin\alpha - N_9 - \frac{P_{CH}}{2} = 0,$$

$$N_9 = -0{,}529 \text{ kN}.$$

Table of obtained results

№ bar	Efforts by the method of joints	Efforts by the method of sections	Efforts under snow loads
N_7	−8,852 kH	−8,852 kH	−0,249 kH
N_9	7,005 kH	7,005 kH	−0.259 kH
N_{12}	−24.588 kH	−24.588 kH	−0.324 kH

Conclusion: the values of the forces in the three bars of the truss, found by the method of joints and by the method of compatibility sections, coincide to a hundredth of a kilo Newton.

Problem 2.2.

For a given flat truss (Fig. 2.56), perform the same actions as clauses 1 - 5, under the conditions of example 1, if $F_1 = 10$ kN, $F_2 = 20$ kN, $F_3 = 10$ kN, and also determine the deformations of the marked bars of the truss.

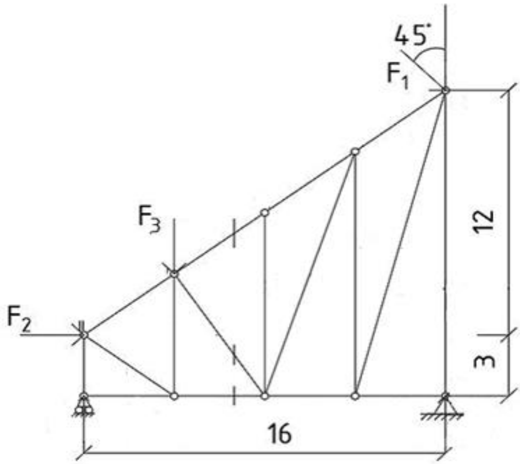


Fig. 2.56. The given truss in Problem 2.2.

Solution:

1. Kinematic analysis. The given calculation scheme contains of 18 bars, 10 joints, a hinge and one disc. (Fig. 2.57).

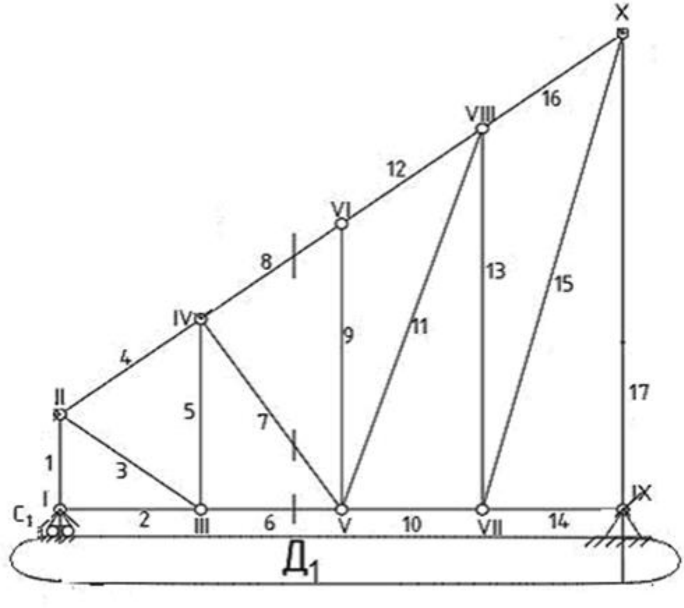


Fig. 2.57. The calculation scheme of truss in Problem 2.2.

1.1. Quantitative analysis.

$Д = 1, \quad B = 10, \quad Ш = 1, \quad C = 18.$

Then, according to the Chebyshov's formula, we get:

$$Г = 3 \cdot 1 + 10 \cdot 2 - 2 \cdot 1 - 18 - 3 = 23 - 23 = 0.$$

Conclusion: the given construction is statically defenitely system.

1.2. Qualitative analys. The given calculation scheme is built in 10 stages, of which 1 - 9 are the Dyad method; 10 – Polonzo's method.

Thus, the given construction is geometrically invariable system.

2. Determination of reactions of supports (Fig. 2.58):

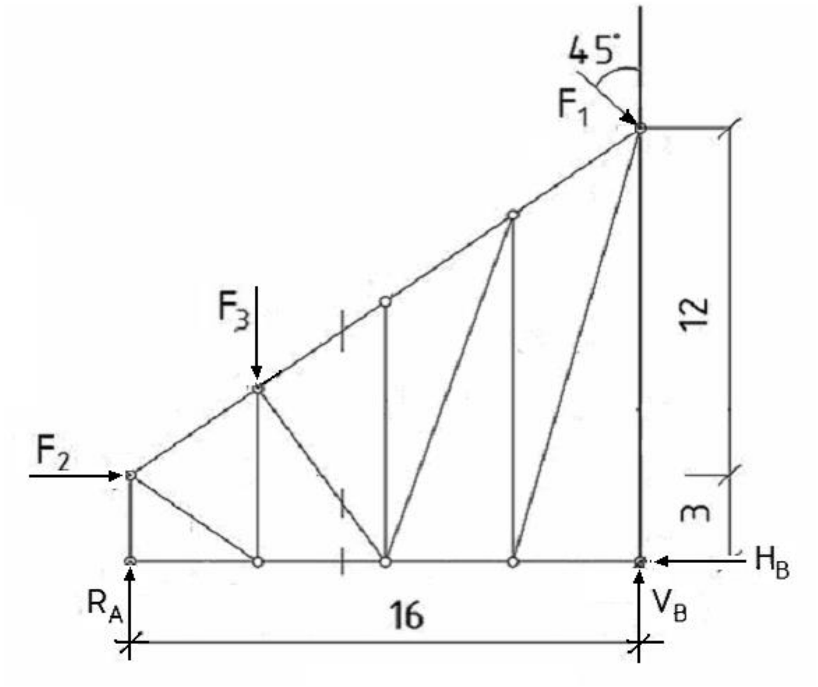


Fig. 2.58. The truss in Problem 2.2 with reactions.

1. $\Sigma F_{x_i} = 0,\qquad F_1 \cdot \sin 45^0 - H_B + F_2 = 0,$

$H_B = F_1 \cdot \sin 45^0 + F_2,\ H_B = 10 \cdot 0{,}707 + 20 = 27{,}07$ kN.

2. $\sum M_{A_i} = 0,$

$$V_B \cdot 16 - F_1 \cdot 16 \cdot \cos 45^0 - F_1 \cdot 15 \cdot \sin 45^0 - F_2 \cdot 3 - F_3 \cdot 4 = 0,$$

$$V_B = \frac{F_1 \cdot 16 \cdot \cos 45^0 + F_1 \cdot 15 \cdot \sin 45^0 + F_2 \cdot 3 + F_3 \cdot 4}{16},$$

$$V_B = \frac{10 \cdot 0{,}707 \cdot 31 + 20 \cdot 3 + 30 \cdot 4}{16} = 24{,}95 \text{ kN.}$$

3. $\sum M_{B_i} = 0, \; -R_A \cdot 16 - F_1 \cdot 15 \cdot \cos 45^0 - F_2 \cdot 3 + F_3 \cdot 12 = 0,$

$$R_A = \frac{-F_1 \cdot 15 \cdot \cos 45^0 - F_2 \cdot 3 + F_3 \cdot 12}{16},$$

$$R_A = \frac{-10 \cdot 15 \cdot \cos 45^0 - 20 \cdot 3 + 30 \cdot 12}{16} = 12{,}12 \text{ kN.}$$

We will check the found reactions:

$$\sum F_{y_i} = 0, \quad R_A + V_B - F_1 \cdot \cos 45^0 - F_3 =$$
$$= 12{,}12 + 24{,}95 - 10 \cdot 0{,}707 - 30 = 37{,}07 - 37{,}07 = 0.$$

Thus, the reactions were found correctly.

3. We are going to determine the desired forces in the bars by the method of joints.

3.1. We will consider the equilibrium of joint I (Fig. 2.59)

$$\sum F^I_{x_i} = 0, \quad N_2 \equiv 0,$$

$$\sum F^I_{y_i} = 0, \quad R_A + N_1 = 0,$$

$$N_1 = -R_A = -12{,}12 \text{ kN.}$$

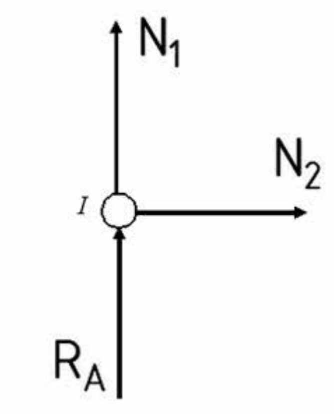


Fig. 2.59. The equilibrium of joint I.

3.2. Let us consider the equilibrium of the joint *II* (Fig. 2.60).

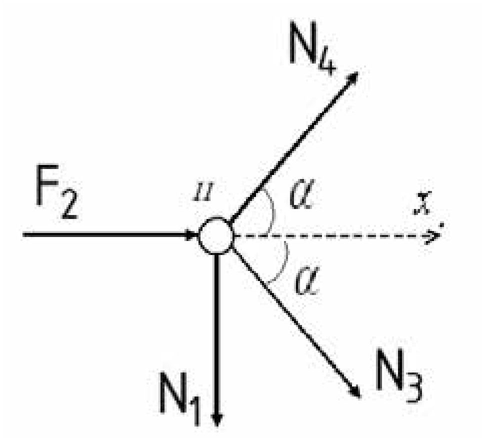


Fig. 2.60. The equilibrium of joint *II*.

From the geometry of the truss, we determine the characteristics of the angle α:

$$\sin\alpha = 0{,}6, \qquad \cos\alpha = 0{,}8.$$

We have to write down the equilibrium equation for the joint II.
Then we get from the equilibrium equations:

$$\sum F_{y_i}^{II} = 0, \quad -N_1 + N_4 \cdot \sin\alpha - N_3 \cdot \sin\alpha = 0,$$

$$\sum F_{x_i}^{A} = 0, \quad F_2 + N_3 \cdot \cos\alpha + N_4 \cdot \cos\alpha = 0,$$

From the last equation we find that:

$$N_3 = -\frac{F_2 + N_4 \cdot \cos\alpha}{\cos\alpha}.$$

Then, substituting the expression for effort N_3 into the first equation, we get:

$$-N_1 + N_4 \cdot \sin\alpha + \frac{F_2 + N_4 \cdot \cos\alpha}{\cos\alpha} \sin\alpha = 0,$$

or

$$-N_1 + 2N_4 \cdot \sin\alpha + \frac{F_2 \sin\alpha}{\cos\alpha} = 0,$$

where

$$N_4 = \frac{N_1}{2\sin\alpha} - \frac{F_2}{2\cos\alpha}.$$

We finally get that:

$$N_4 = \frac{-12{,}12}{2 \cdot 0{,}6} - \frac{20}{2 \cdot 0{,}8} = -22{,}6 \text{ kN}.$$

Respectively we obtain that:

$$N_3 = -\frac{20 - 22{,}6 \cdot 0{,}8}{0{,}8} = -2{,}4 \text{ kN}.$$

3.3. Let us consider the equilibrium of joint *III* (Fig. 2.61).

$\Sigma F_{x_i}^{III} = 0, \ -N_2 + N_6 - N_3 \cos\alpha = 0,$

$N_6 = N_2 + N_3 \cos\alpha,$

or

$N_6 = -2{,}4 \cdot 0{,}8 = -1{,}92$ kN;

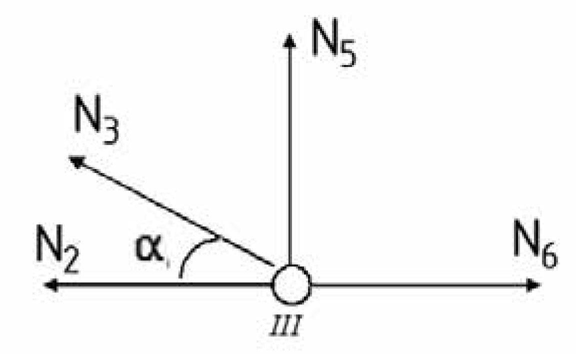


Fig. 2.61. The equilibrium of joint *III*.

$\Sigma F_{y_i}^{III} = 0, \ N_5 + N_3 \cdot \sin\alpha = 0,$

$N_5 = -N_3 \cdot \sin\alpha,$

or

$N_5 = 2{,}4 \cdot 0{,}6 = 1{,}44$ kN.

3.4. Let us consider the equilibrium of joint *IV* (рис. 2.62).

$$\Sigma F_{x_i}^{II} = 0, \qquad N_7 \cdot \cos\beta + N_8 \cdot \cos\alpha - N_4 \cdot \cos\alpha = 0, \qquad (2.14)$$

$$\Sigma F_{y_i}^{III} = 0, - N_7 \cdot \sin\beta + N_8 \cdot \sin\alpha - F_3 - N_5 - N_4 \cdot \sin\alpha = 0, \qquad (2.15)$$

From the geometry of the truss, we determine the characteristics of the angle β:

$$\sin\beta = 0{,}832, \quad \cos\beta = 0{,}554.$$

After solving the system of equations (2.14) and (2.15), we obtain the value of the force in bars 7 and 8:

$$N_7 = -25{,}2 \text{ kN}, \qquad N_8 = -5{,}1 \text{ kN}. \qquad (2.16)$$

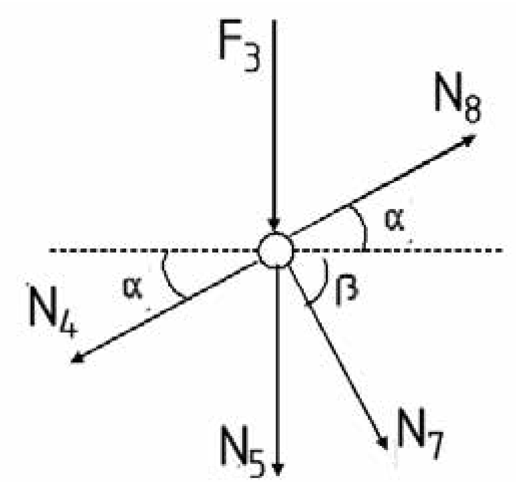


Fig. 2.62. The equilibrium of joint *IV*.

3. We will check the found forces in bars 7th, 9th and 12th, using the Ritter's method.

Let us consider the Ritter's section (Fig. 2.63):

$$\Sigma M_{V_i} = 0, \; -N_8 \cdot 6 \cdot \cos\alpha - N_8 \cdot 4 \cdot \sin\alpha + F_3 \cdot 4 - F_2 \cdot 3 - R_A \cdot 8 = 0,$$

or

$$-N_8 \cdot 6 \cdot 0{,}8 - N_8 \cdot 4 \cdot 0{,}6 + 30 \cdot 4 - 20 \cdot 3 - 12{,}12 \cdot 8 = 0,$$

where:

$$N_8 = \frac{30 \cdot 4 - 20 \cdot 3 - 12{,}12 \cdot 8}{6 \cdot 0{,}8 + 4 \cdot 0{,}6} = \frac{-36{,}96}{7{,}2} \approx -5{,}1 \text{ kN}. \qquad (2.17)$$

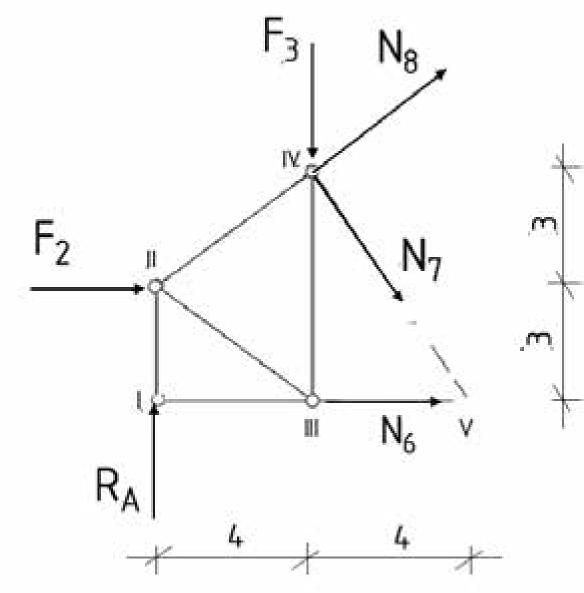


Fig. 2.63. The Ritter's section for truss of problem 2.2.

$$\Sigma M_{IV_i} = 0, \qquad F_2 \cdot 3 - R_A \cdot 4 + N_6 \cdot 6 = 0$$

where:

$$N_6 = \frac{R_A \cdot 4 - F_2 \cdot 3}{6},$$

or

$$N_6 = \frac{12{,}12 \cdot 4 - 20 \cdot 3}{6} = -1{,}92 \text{ kN}. \qquad (2.18)$$

$$\Sigma F_{x_i} = 0, \quad N_8 \cdot \cos\alpha + N_7 \cdot \cos\beta + N_6 + F_2 = 0,$$

where:

$$N_7 = -\frac{N_8 \cdot \cos\alpha + N_6 + F_2}{\cos\beta},$$

or:

$$N_7 = -\frac{-5{,}1 \cdot 0{,}8 - 1{,}92 + 20}{0{,}554} = -25{,}2 \text{ kN}. \qquad (2.19)$$

If we compare expressions (2.16) and expressions (2.17) - (2.19), it can be seen that the corresponding values of the required forces in the truss bars coincide to the first sign after the decimal point.

5. The determination of forces in truss bars under snow load.

Let a snow load with an intensity of $q = 0{,}8$ kN/m^2 be applied to the upper belt of the given truss (Fig. 2.64).

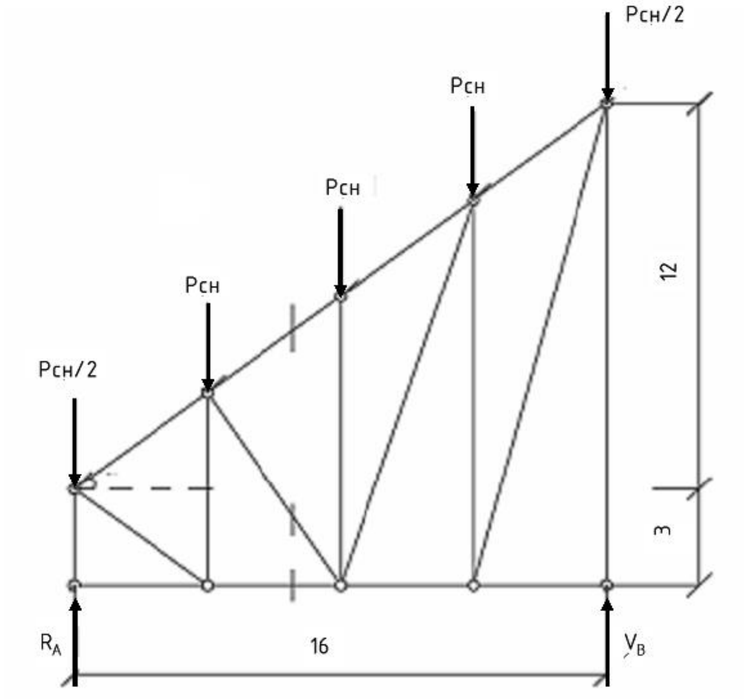


Fig. 2.64. The snow load is applied to the upper belt of the given truss.

We are going to determine the forces in bars 6th, 7th and 8th of the given truss under the snow load.

5.1. Given that $\alpha = 36{,}87^0$, we will find the coefficient c for calculating the nodal snow load according to formula (2.1). Since $30^0 \le \alpha \le 45^0$, the value of the coefficient c will be in the range from 1 to 5. To calculate it, we will use the interpolation method:

$$c = 1 - \frac{0{,}5}{15} \cdot 6{,}87 = 0{,}771. \qquad (2.20)$$

Then according to formula (2.1) we get:

$$P_{CH} = 1{,}4 \cdot 0{,}8 \cdot 0{,}771 \cdot 4 \cdot 0{,}5 = 1{,}727 \text{ kN}.$$

5.2. We apply P_{CH} to the joints of the upper belt of the truss according to fig. 2.64 and determine the support reactions:

1. $\Sigma M_{A_i} = 0, \quad V_B \cdot 16 - P_{CH} \cdot (8 + 12 + 8 + 4) = 0$

$$V_B = \frac{P_{CH} \cdot 32}{16} = 2P_{CH}.$$

2. $\Sigma M_{B_i} = 0, \; -R_A \cdot 16 + P_{CH} \cdot (8 + 12 + 8 + 4) = 0$

$$V_A = \frac{P_{CH} \cdot 32}{16} = 2P_{CH}.$$

We will check the found reactions:

$$\Sigma F_{y_i} = 0, \quad R_A + V_B - P_{CH} \cdot 3 - \frac{P_{CH}}{2} - \frac{P_{CH}}{2} = 2P_{CH} + 2P_{CH} - P_{CH} \cdot 4 =$$
$$= 4P_{CH} - 4P_{CH} = 0.$$

So, the reactions are found correctly.

5.3. Using Ritter's method, we determine the forces in the marked bars of the truss under the snow load.

Let us run the Ritter’s section (Fig. 2.65).

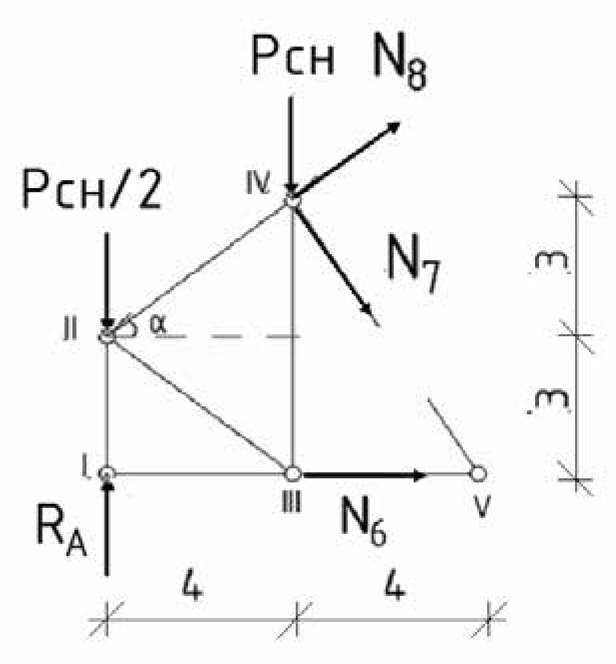


Fig. 2.65. The Ritter’s section of truss under snow load.

$$\Sigma M_{V_i} = 0, \quad -N_8 \cdot 6 \cdot \cos\alpha - N_8 \cdot 4 \cdot \sin\alpha - R_A \cdot 8 + P_{CH} \cdot 4 + \frac{P_{CH}}{2} \cdot 8 = 0,$$

where

$$N_8 = \frac{-R_A \cdot 8 + P_{CH} \cdot 8}{6 \cdot \cos\alpha + 4 \cdot \sin\alpha},$$

or:

$$N_8 = \frac{-2P_{CH} \cdot 8 + P_{CH} \cdot 8}{6 \cdot 0{,}8 + 4 \cdot 0{,}6} = -\frac{P_{CH} \cdot 8}{7{,}2} = -1{,}1P_{CH} = -1{,}92 \text{ kN}.$$

$$\sum M_{IV_i} = 0, \quad N_6 \cdot 6 - R_A \cdot 4 + \frac{P_{CH}}{2} \cdot 4 = 0,$$

where

$$N_6 = \frac{R_A \cdot 4 - P_{CH} \cdot 2}{6} = \frac{2P_{CH} \cdot 4 - P_{CH} \cdot 2}{6} = \frac{P_{CH} \cdot 6}{6} = P_{CH},$$

or:

$$N_6 = 1{,}727 \text{ kN}.$$

$$\sum F_{x_i} = 0, \; N_8 \cdot \cos\alpha + N_7 \cos\beta + N_6 = 0,$$

where:

$$N_7 = -\frac{N_8 \cdot \cos\alpha + N_6}{\cos\beta},$$

or

$$N_7 = -\frac{-1{,}92 \cdot 0{,}8 + 1{,}727}{0{,}554} = -0{,}345 \text{ kN}.$$

Table of obtained results

№ bar	Efforts by the method of joints	Efforts by the method of sections	Efforts under snow loads
N_6	-1,92 kN	-1,92 kN	1,727 kN
N_7	-25,2 kN	-25,2 kN	-0,345 kN
N_8	-5,1 kN	-5,1 kN	-1,92 kN

Conclusion: the values of the forces in the three bars of the truss, found by the method of joints and by the method of compatibility sections, coincide to a hundredth of a kilo newton.

6. We are going to determine the deformations in truss bars 6th, 7th and 8th from the technological load, if $E = 2 \cdot 10^5$ MPa

The bar of truss works in compression or tension. As you know, from the course of mechanics of materials and structures (strength of materials), the deformation of the bar during tension-compression is determined by the formula:

$$\Delta\ell_i = \frac{N_i \cdot \ell_i}{EA}, \qquad (2.21)$$

where A is the cross-sectional area of the bar.

According to the conditions of the problem, the truss bars have a circular cross-section. In this case, the diameter of the bar is equal to 0.5 m. Then the cross-sectional area will be equal to:

$$A = \frac{\pi d^2}{4}, \quad \text{or} \quad A = \frac{3{,}14 \cdot 0{,}5^2}{4} = 0{,}196 \text{ m}^2.$$

Then we calculate the corresponding deformations of truss bars 6th, 7th and 8th according to formula (2.21):

$$\Delta\ell_6 = \frac{-1{,}92 \cdot 10^3 \cdot 4}{2 \cdot 10^{11} \cdot 0{,}196} = -19{,}6 \cdot 10^{-5} \text{ mm};$$

$$\Delta\ell_7 = \frac{-25{,}2 \cdot 10^3 \cdot 7{,}2}{2 \cdot 10^{11} \cdot 0{,}196} = -462{,}9 \cdot 10^{-5} \text{ mm};$$

$$\Delta\ell_8 = \frac{-5{,}1 \cdot 10^3 \cdot 5}{2 \cdot 10^{11} \cdot 0{,}196} \approx -65 \cdot 10^{-5} \text{ mm}.$$

The obtained results indicate that all three bars of given truss will work on tension.

Problem 2.3.

For a given flat truss (Fig. 2.66), perform the same actions as clauses 1 - 6, under the conditions of problem 2.2, if $F_1 = 10$ kN, $F_2 = 20$ kN, $F_3 = 10$ kN, and also determine the deformations of the marked bars of the truss.

Solution:

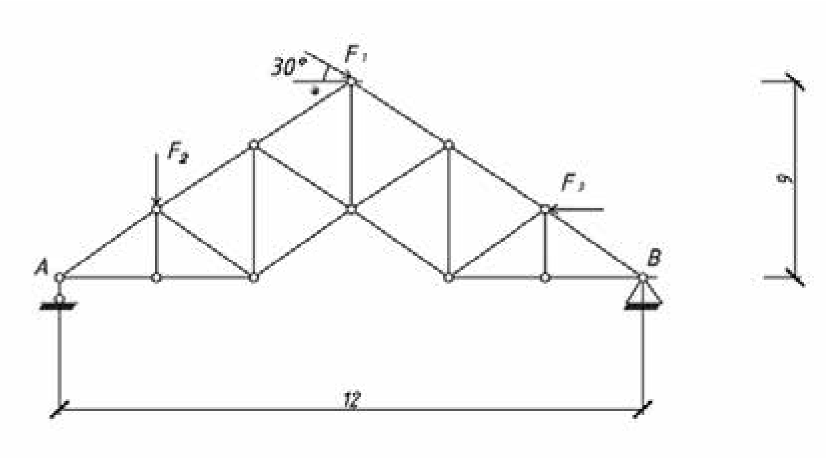


Fig. 2.66. The given truss in Problem 2.3.

1. Kinematic analysis. The given calculation scheme contains of 18 bars, 10 joints, a hinge and one disc. (Fig. 2.67).

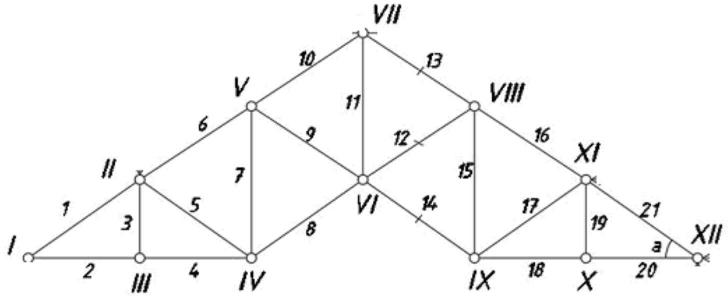


Fig. 2.67. The calculation scheme of truss in Problem 2.3.

1.1. Quantitative analysis.

$Д = 1, \qquad В = 12, \qquad Ш = 1, \qquad C = 22.$

Then, according to the Chebyshov's formula, we get:

$$\Gamma = 3 \cdot 1 + 12 \cdot 2 - 2 \cdot 1 - 22 - 3 = 27 - 27 = 0.$$

Conclusion: the given construction is statically definitely system.

1.2. Qualitative analys. The given calculation scheme is built in 12 stages, of which 1 - 11 are the Dyad method; 10 – Polonzo's method.

Thus, the given construction is geometrically invariable system.

2. The definition of support reactions of given (Fig. 2.68):

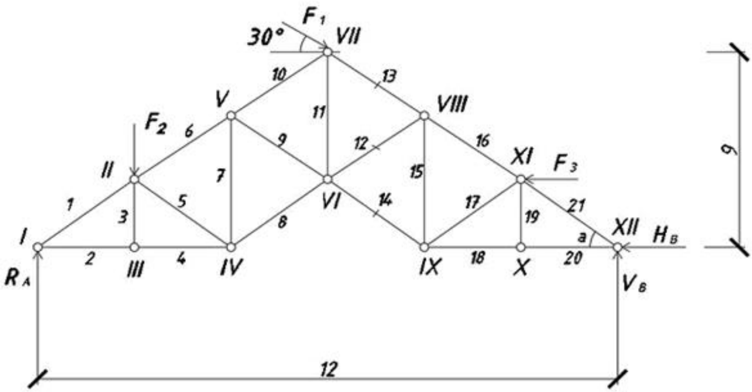


Fig. 2.68. The given truss with reactions in Problem 2.3.

1. $\sum F_{x_i} = 0, \qquad F_1 \cdot \cos 30^0 - H_B - F_3 = 0,$

$H_B = F_1 \cdot \cos 30^0 - F_3, H_B = 10 \cdot 0{,}866 - 30 = -21{,}34$ kN.

2. $\sum M_{A_i} = 0,$

$$V_B \cdot 12 - F_1 \cdot 9 \cdot \cos 30^0 - F_1 \cdot 6 \cdot \sin 30^0 - F_2 \cdot 2 + F_3 \cdot 3 = 0,$$

$$V_B = \frac{F_1 \cdot 9 \cdot \cos 30^0 + F_1 \cdot 6 \cdot \sin 30^0 + F_2 \cdot 2 - F_3 \cdot 3}{12},$$

$$V_B = \frac{10 \cdot 9 \cdot 0{,}866 + 10 \cdot 6 \cdot 0{,}5 + 20 \cdot 2 - 30 \cdot 3}{12} = 4{,}828 \text{ kN}.$$

3. $\Sigma M_{B_i} = 0,$

$$-R_A \cdot 12 - F_1 \cdot 9 \cdot \cos 30^0 + F_1 \cdot 6 \cdot \sin 30^0 + F_2 \cdot 10 + F_3 \cdot 3 = 0,$$

$$R_A = \frac{-F_1 \cdot 9 \cdot \cos 30^0 + F_1 \cdot 6 \cdot \sin 30^0 + F_2 \cdot 10 + F_3 \cdot 3}{12},$$

$$R_A = \frac{-10 \cdot 9 \cdot 0{,}866 + 10 \cdot 6 \cdot 0{,}5 + 20 \cdot 10 + 30 \cdot 3}{16} = 20{,}172 \text{ kN}.$$

The verification of found reactions:

$$\Sigma F_{y_i} = 0, \quad R_A + V_B - F_1 \cdot \sin 30^0 - F_2 =$$
$$= 20{,}172 + 4{,}828 - 10 \cdot 0{,}5 - 20 = 25 - 25 = 0.$$

Thus, the reactions of supports were found correctly.

3. We are going to find the efforts of truss bars by method of joints.
3.1. Let us consider the equilibrium of joint XII (Fig. 2.69).

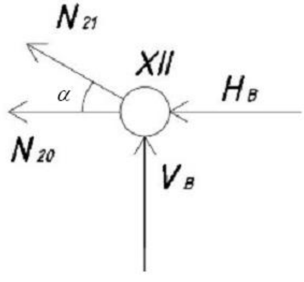


Fig. 2.69. The equilibrium of joint XII.

From the geometry of the truss, we determine the characteristics of the angle α:

$$\cos\alpha = 0{,}5547, \quad \sin\alpha = 0{,}8321$$

Then we can write:

$$\sum F_{y_i}^{XII} = 0, \qquad -N_{21} \cdot \sin\alpha - V_B = 0,$$

where:

$$N_{21} = -\frac{V_B}{\sin\alpha},$$

or

$$N_{21} = -\frac{4{,}828}{0{,}8321} = -5{,}802 \text{ kN}.$$

$$\sum F_{x_i}^{XII} = 0, \qquad H_B + N_{20} + N_{21}\cos\alpha = 0,$$

where:

$$N_{20} = -H_B - N_{21}\cos\alpha$$

or

$$N_{20} = 21{,}34 + 8{,}803 \cdot 0{,}5547 = 24{,}559 \text{ kN}.$$

3.2. Let us consider the equilibrium of joint X (Fig. 2.70):

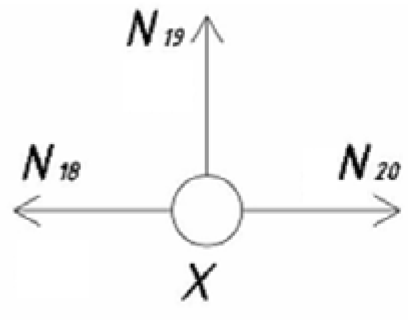


Fig. 2.70. The equilibrium of joint X.

Let us write the equilibrium equation for the joint X. Then we get from the equilibrium equations:

$$\sum F_{y_i}^{X} = 0, \quad N_{19} \equiv 0,$$

$$\sum F_{x_i}^{X} = 0, \quad N_{20} - N_{18} = 0,$$

where:

$$N_{18} = N_{20} = 24{,}559 \text{ kN}.$$

3.3. Let us consider the equilibrium of joint XI (Fig. 2.71)

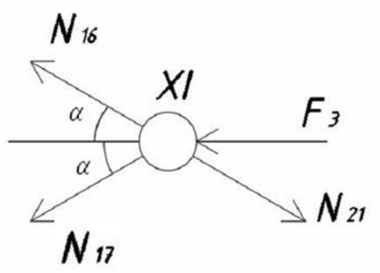


Fig. 2.71. The equilibrium of joint XI.

$$\sum F_{x_i}^{XI} = 0, \quad N_{21} \cos\alpha - N_{16} \cos\alpha - N_{17} \cos\alpha - F_3 = 0, \tag{2.22}$$

$$\sum F_{y_i}^{XI} = 0, \quad N_{16} \cdot \sin\alpha - N_{17} \cdot \sin\alpha - N_{21} \cdot \sin\alpha = 0, \tag{2.23}$$

From equation (2.23) we obtain that:

$$N_{17} = N_{16} - N_{21}. \tag{2.24}$$

Substituting expression (2.24) into equation (2.22), we get:

$$N_{16} = N_{21} - \frac{F_3}{2\cos\alpha}.$$

Then:

$$N_{16} = -5{,}802 - \frac{30}{2 \cdot 0{,}5547} = -32{,}844 \text{ kN}.$$

$$N_{17} = -32{,}844 + 5{,}802 = -27{,}042 \text{ kN}.$$

3.4. Let us consider the equilibrium of joint IX (Fig. 2.72).

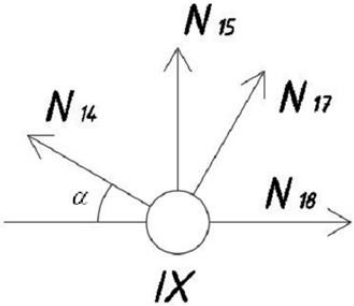


Fig. 2.72. The equilibrium of joint IX.

$$\Sigma F_{x_i}^{IX} = 0, \qquad N_{17} \cdot \cos\alpha + N_{18} - N_{14} \cdot \cos\alpha = 0,$$

where:

$$N_{14} = \frac{N_{17} \cdot \cos\alpha + N_{18}}{\cos\alpha},$$

or:

$$N_{14} = \frac{24{,}559 - 27{,}042 \cdot 0{,}5547}{0{,}5547} = 17{,}232 \text{ kN}; \qquad (2.25)$$

$$\Sigma F_{y_i}^{IX} = 0, \qquad -N_{17} \cdot \sin\alpha - N_{15} - N_{14} \cdot \sin\alpha = 0,$$

where:

$$N_{15} = -(N_{17} + N_{14}) \cdot \sin\alpha,$$

or:

$$N_{15} = -(17{,}232 - 27{,}042) \cdot 0{,}8321 = 8{,}163 \text{ kN}.$$

3.5. Let us consider the equilibrium of joint VIII (Fig. 2.73).

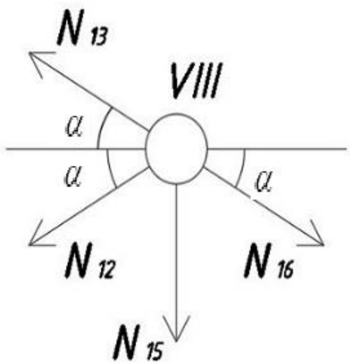


Fig. 2.73. The equilibrium of joint VIII.

$$\Sigma F_{x_i}^{VIII} = 0, \qquad N_{16}\cos\alpha - N_{13}\cos\alpha - N_{12}\cos\alpha = 0, \qquad (2.26)$$

$$\Sigma F_{y_i}^{VIII} = 0, \quad N_{15} + N_{16}\cdot\sin\alpha - N_{13}\cdot\sin\alpha + N_{12}\cdot\sin\alpha = 0, \quad (2.27)$$

From equation (2.26) we find the expression for the force N_{12}:

$$N_{12} = N_{16} - N_{13}. \qquad (2.28)$$

Then, if we substituted the expression (2.28) into equation (2.27), we get:

$$N_{15} + N_{16}\cdot\sin\alpha - N_{13}\cdot\sin\alpha + (N_{16} - N_{13})\cdot\sin\alpha = 0,$$

where:

$$N_{13} = N_{16} + \frac{N_{15}}{2\sin\alpha},$$

or:

$$N_{13} = -32{,}843 + \frac{8{,}163}{2 \cdot 0{,}8321} = -27{,}939 \text{ kN}. \tag{2.29}$$

Then we get that:

$$N_{12} = -32{,}844 + 27{,}939 = -4{,}905 \text{ kN}. \tag{2.30}$$

4. We will check the forces found in bars 13th and 14th using the method of section.

Let us consider the Ritter's section (Fig. 2.74):

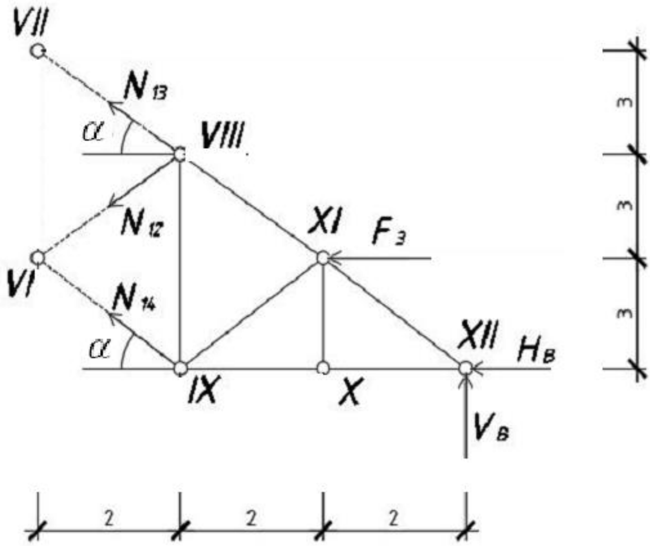


Fig. 2.74. The Ritter's section for truss of problem 2.3.

$$\sum M_{VIII_i} = 0, \; -N_{14} \cdot 6 \cdot \cos\alpha + V_B \cdot 4 - H_B \cdot 6 - F_3 \cdot 3 = 0,$$

where:

$$N_{14} = \frac{V_B \cdot 4 - H_B \cdot 6 - F_3 \cdot 3}{6 \cdot \cos\alpha},$$

or:

$$N_{14} = \frac{4{,}828 \cdot 4 + 21{,}34 \cdot 6 - 30 \cdot 3}{6 \cdot 0{,}8321} = 17{,}232 \text{ kN}. \qquad (2.31)$$

$$\Sigma M_{VI_i} = 0, \qquad V_B \cdot 6 - H_B \cdot 3 - N_{13} \cdot 2 \cdot \sin\alpha + N_{13} \cdot 3 \cdot \cos\alpha = 0,$$

where:

$$N_{13\cdot} = \frac{H_B \cdot 3 - V_B \cdot 6}{3 \cdot \cos\alpha - 2 \cdot \sin\alpha},$$

or

$$N_{13\cdot} = \frac{-21{,}34 \cdot 3 - 4{,}828 \cdot 6}{3 \cdot 0{,}8321 - 2 \cdot 0{,}5547} = -27{,}939 \text{ kN}. \qquad (2.32)$$

If we compare the expressions (2.25) and (2.29) with the expressions (2.31) and (2.32), respectively, it can be seen that the corresponding values of the required forces in the truss bars coincide to the third decimal place.

5. The determination of forces in truss bars under snow load.

Let a snow load with an intensity of $q = 0{,}7$ kN/m^2 be applied to the upper belt of the given truss (Fig. 2.75).

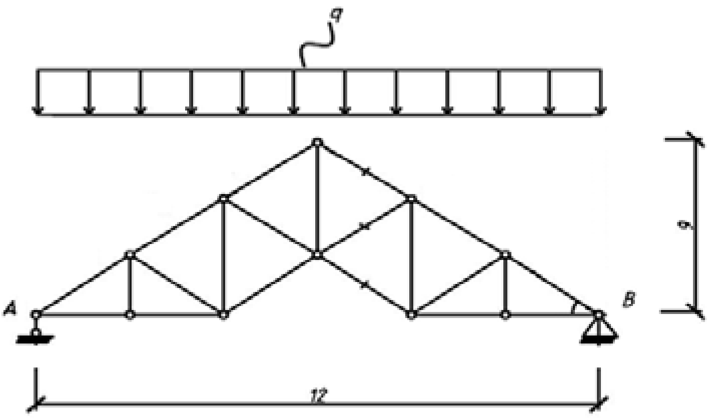


Fig. 2.75. The truss under snow load in problem 2.3.

We are going to determine the forces in bars 6th, 7th and 8th of the given truss under the snow load.

5.1. Given that $\alpha = 56{,}3^0$, we will find the coefficient c for calculating the nodal snow load according to formula (2.1). Since $45^0 \le \alpha \le 60^0$, the value of the coefficient c will be in the range from 0,5 to 0. To calculate it, we will use the interpolation method:

$$c = 0{,}5 - \frac{0{,}5}{15} \cdot 11{,}3 = 0{,}123. \qquad (2.33)$$

Then according to formula (2.1) we get:

$$P_{CH} = 1{,}4 \cdot 0{,}7 \cdot 0{,}123 \cdot 2 \cdot 0{,}4 = 0{,}096 \text{ kN}.$$

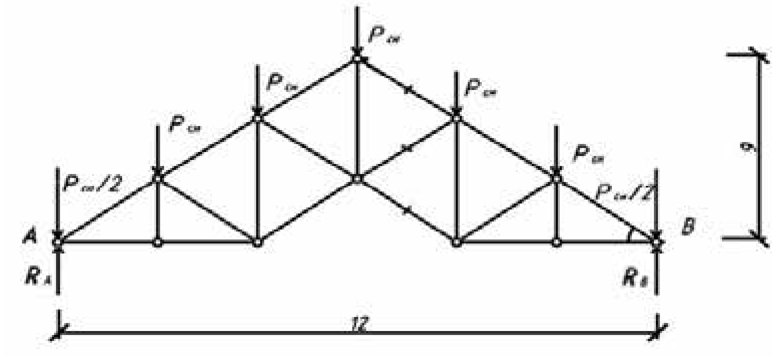


Fig. 2.76. The joint snow load is on upper belt of the truss.

5.2. We apply forces P_{CH} to the joints of the upper belt of the truss according to fig. 2.76 and we are going to determine the support reactions of this truss:

1. $\sum M_{A_i} = 0, \qquad R_B \cdot 12 - P_{CH} \cdot (6 + 12 + 8 + 6 + 4 + 2) = 0$

$$R_B = \frac{P_{CH} \cdot 36}{12} = 3P_{CH}.$$

2. $\sum M_{B_i} = 0,\ -R_A \cdot 12 + P_{CH} \cdot (6+12+8+6+4+2) = 0$

$$R_A = \frac{P_{CH} \cdot 36}{12} = 3P_{CH}.$$

We will check the found reactions:

$$\sum F_{y_i} = 0, \quad R_A + R_B - P_{CH} \cdot 5 - \frac{P_{CH}}{2} - \frac{P_{CH}}{2} = 3P_{CH} + 3P_{CH} - P_{CH} \cdot 6 =$$
$$= 6P_{CH} - 6P_{CH} = 0.$$

So, the reactions are found correctly.

5.3. Using method of sections, we will determine the forces in the marked bars of the truss under the snow load.

Let us run the Ritter's section (Fig. 2.77).

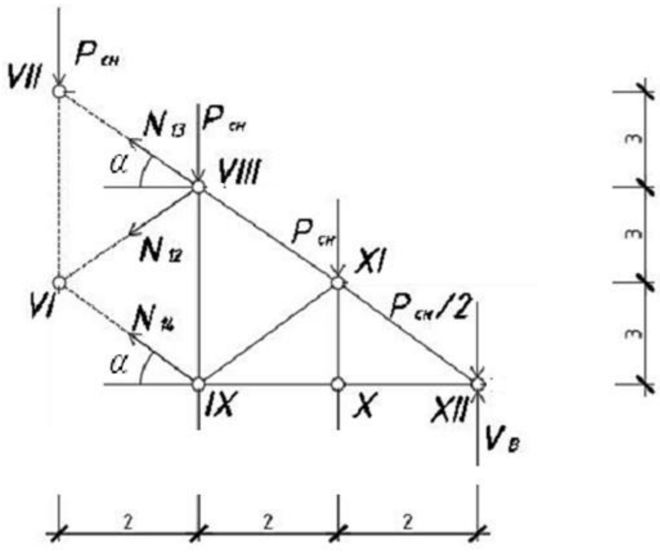


Fig. 2.77. The Ritter's section of the truss under snow load.

$\sum M_{VIII_i} = 0$, $R_B \cdot 4 - P_{CH} \cdot 2 - P_{CH} \cdot 2 - N_{14} \cdot 6 \cdot \cos\alpha = 0$,
where

$$N_{14} = \frac{R_B \cdot 4 - P_{CH} \cdot 4}{6 \cdot \cos\alpha},$$

or:

$$N_{14} = \frac{3P_{CH} \cdot 4 - P_{CH} \cdot 4}{6 \cdot 0{,}8321} = 2{,}404 P_{CH} = 0{,}231 \text{ kN}.$$

$\sum M_{VI_i} = 0$,

$$R_B \cdot 6 - P_{CH} \cdot 2 - P_{CH} \cdot 3 - P_{CH} \cdot 4 + 2N_{13} \cdot \sin\alpha + 3N_{13} \cdot \cos\alpha = 0,$$

where

$$N_{13} = \frac{P_{CH} \cdot 9 - R_B \cdot 6}{2\sin\alpha + 3\cos\alpha},$$

or:

$$N_{13} = \frac{-P_{CH} \cdot 9}{2 \cdot 0{,}5547 + 3 \cdot 0{,}8321} = -2{,}704 P_{CH}.$$

Finally we get:

$$N_{13} = -2{,}704 \cdot 0{,}096 = 0{,}26 \text{ kN}.$$

$\sum F_{x_i} = 0$, $N_{14} \cdot \cos\alpha + N_{13} \cdot \cos\alpha + N_{12} \cdot \cos\alpha = 0$,
where:

$$N_{12} = -(N_{14} + N_{13}),$$

or

$$N_{12} = -2{,}404 P_{CH} + 2{,}704 P_{CH} = 0{,}3 P_{CH}.$$

Finally we get:

$$N_{12} = 0{,}3 \cdot 0{,}096 = 0{,}029 \text{ kN}.$$

Table of obtained results

№ bar	Efforts by the method of joints	Efforts by the method of sections	Efforts under snow loads
N_{12}	-4,905 kN	-	0,029 kN
N_{13}	-27,939 kN	-27,939 кН	0,26 kN
N_{14}	17,232 kN	17,232 кН	0,231 kN

Conclusion: the values of the forces in the three bars of the truss, found by the method of joints and by the method of sections, coincide to the tenth of a kilo newton.

6. We are going to determine the deformations in truss bars 12th, 13th and 14th from the technological load, if $E = 2 \cdot 10^5$ MPa

The bar of truss works in compression or tension. As you know, from the course of mechanics of materials and structures (strength of materials), the deformation of the bar during tension-compression is determined by the formula:

$$\Delta \ell_i = \frac{N_i \cdot \ell_i}{EA}, \tag{2.21}$$

where A is the cross-sectional area of the bar.

According to the conditions of the problem, the truss bars have a circular cross-section. In this case, the diameter of the bar is equal to 0.5 m. Then the cross-sectional area will be equal to:

$$A = \frac{\pi d^2}{4}, \quad \text{або} \quad A = \frac{3{,}14 \cdot 0{,}4^2}{4} = 0{,}126 \text{ m}^2.$$

If we considering that the lengths of all three bars are the same and equal $\ell = \sqrt{3^2 + 2^2} = \sqrt{13} \approx 3{,}6$ m, than according formula (2.21), we calculate deformation of bars 12th, 13th and 14th of given truss:

$$\Delta\ell_{12} = \frac{-4{,}905 \cdot 10^{3} \cdot 3{,}6}{2 \cdot 10^{11} \cdot 0{,}126} = -70{,}1 \cdot 10^{-5} \text{ mm};$$

$$\Delta\ell_{13} = \frac{-27{,}939 \cdot 10^{4} \cdot 3{,}6}{2 \cdot 10^{11} \cdot 0{,}126} \approx -399 \cdot 10^{-5} \text{ mm};$$

$$\Delta\ell_{14} = \frac{17{,}232 \cdot 10^{4} \cdot 3{,}6}{2 \cdot 10^{11} \cdot 0{,}126} \approx 246 \cdot 10^{-5} \text{ mm}.$$

The obtained results indicate that the truss bars 12th and 13th will work on tension, but bar 14th will compress at a given technological load.

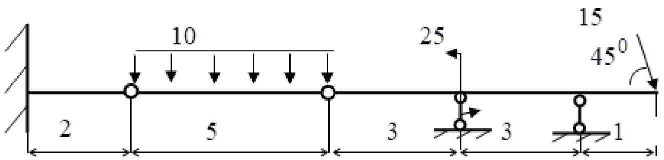
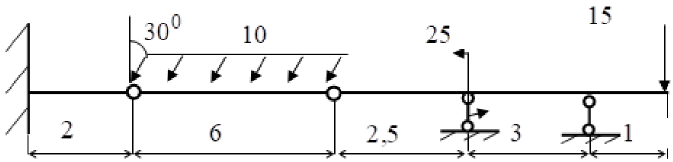
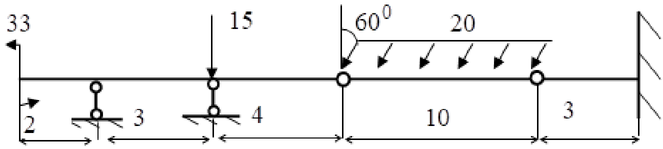
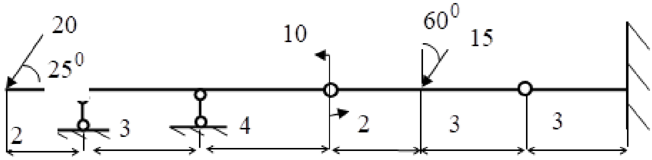
WORD LIST

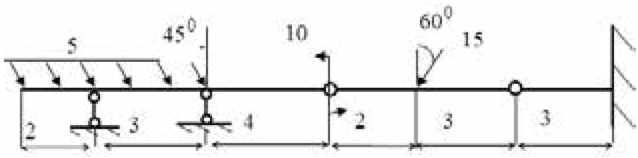
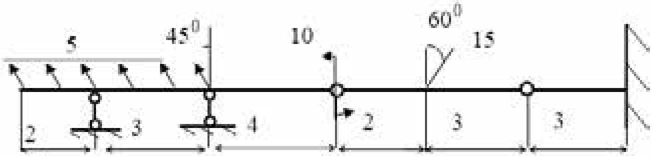
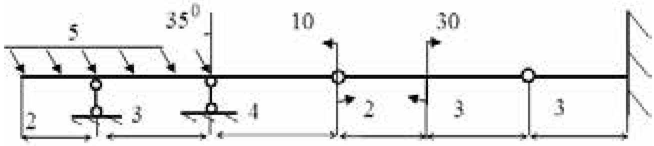
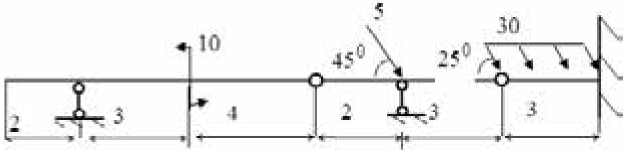
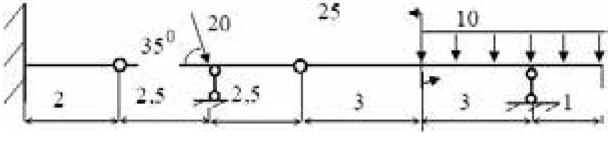
beam	балка
cantilever beam	консольна балка
complex beam	складена балка
cylindrical hinge	циліндричний шарнір
degree of freedom	степінь вільності
DoubI T -beam	двотавр
geometrical variable system	геометрично змінна система
joint	вузол
invariable system	незмінна система
kinematic support	кінетична в’язь
line of influence	лінія впливу
method of joints	метод вирізання вузлів
method of section	метод Ріттера
node	вузол
rigity	жорсткість
simple disk	простий диск
simple supported beam	шарнірна балка
soldering	припайка
truss	ферма

APPENDICE

Individual work 1

Table 1. The Scheme of complex beams

1.	
2.	
3.	
4.	

5.	5 45⁰ 10 60⁰ 15 2 3 4 2 3 3
6.	5 45⁰ 10 60⁰ 15 2 3 4 2 3 3
7.	5 35⁰ 10 30 2 3 4 2 3 3
8.	10 5 45⁰ 25⁰ 30 2 3 4 2 3 3
9.	35⁰ 20 25 10 2 2.5 2.5 3 3 1

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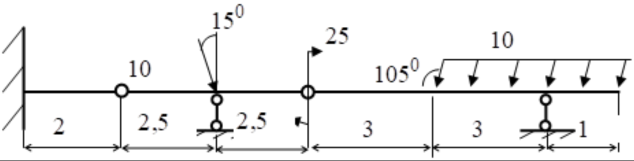

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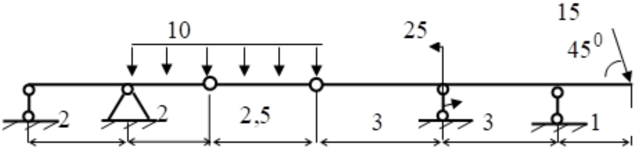

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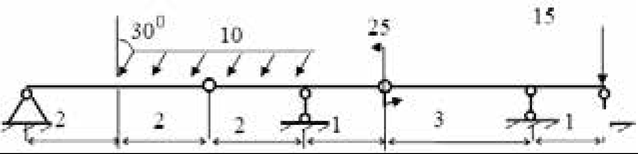

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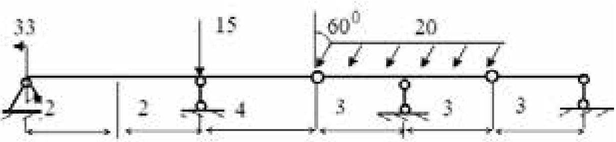

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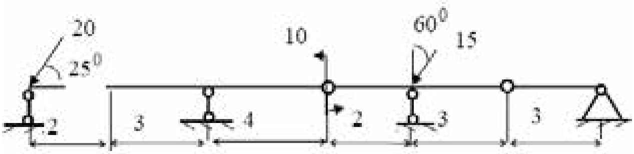

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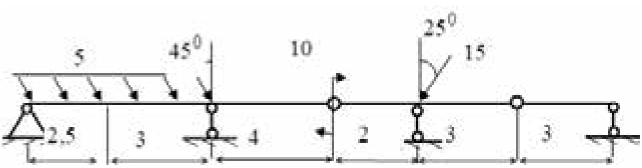
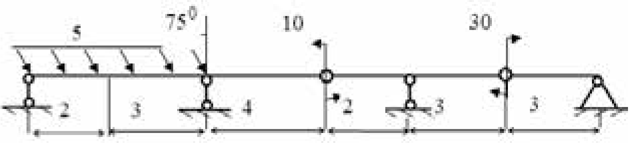
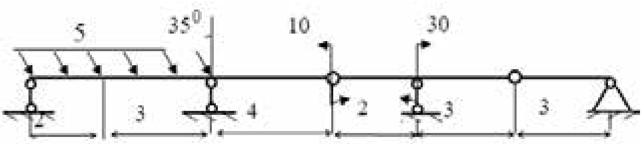
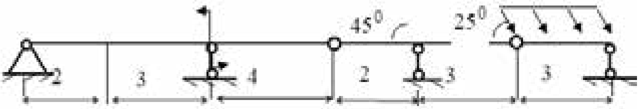
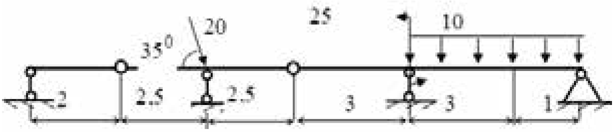
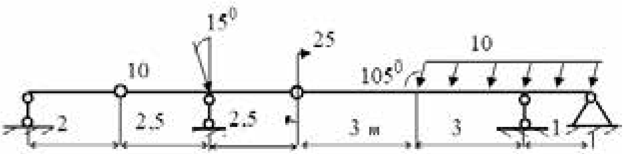
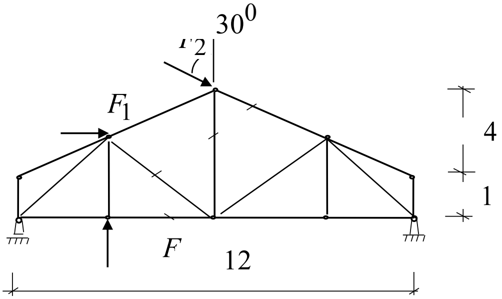


16.	5 75^0 10 30 2 3 4 2 3 3
17.	5 35^0 10 30 2 3 4 2 3 3
18.	45^0 25^0 2 3 4 2 3 3
19.	35^0 20 25 10 2 2.5 2.5 3 3 1
20.	15^0 10 25 10 105^0 2 2.5 2.5 3 м 3 1

Individual work 2

Table 2. The Scheme of trusses

1. $q_{сн} = 0{,}9$ кН/м^2, $a = 0{,}2$ м.



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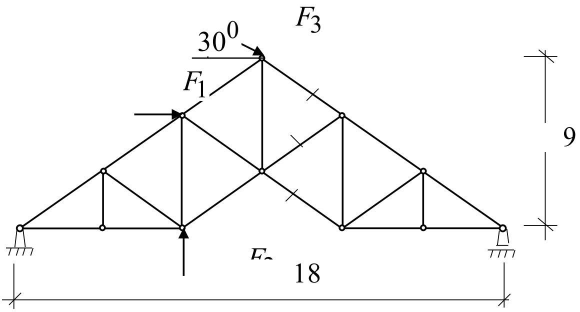

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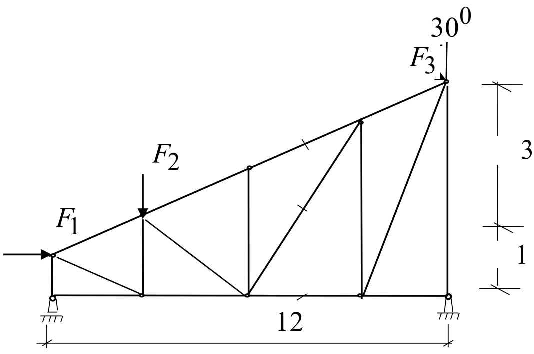

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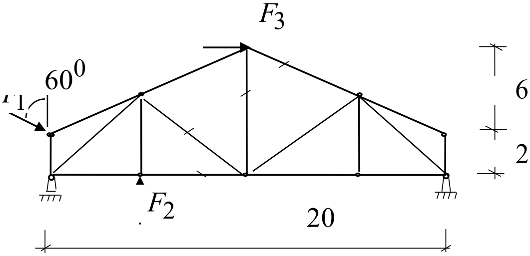</td></tr>
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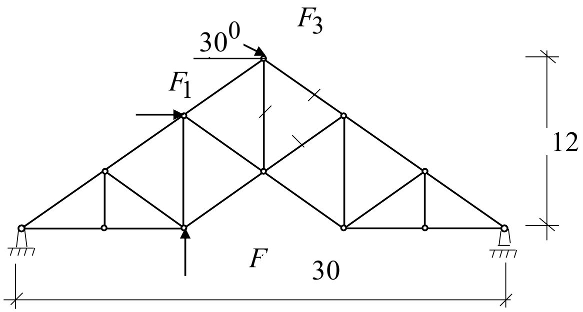</td></tr>
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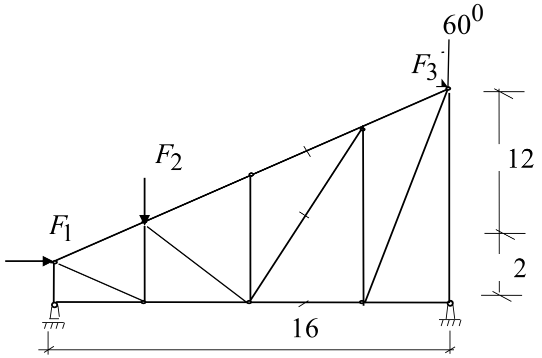</td></tr>
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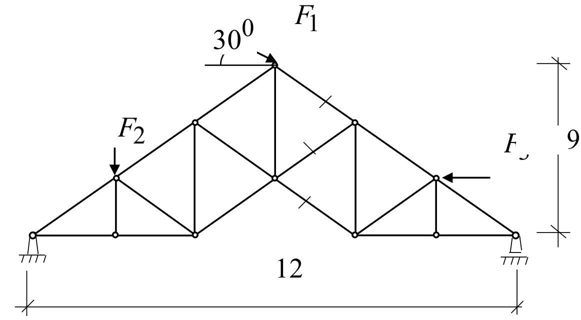
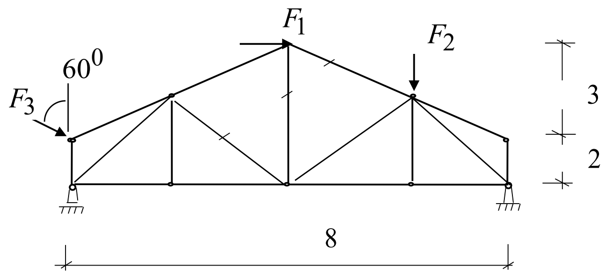
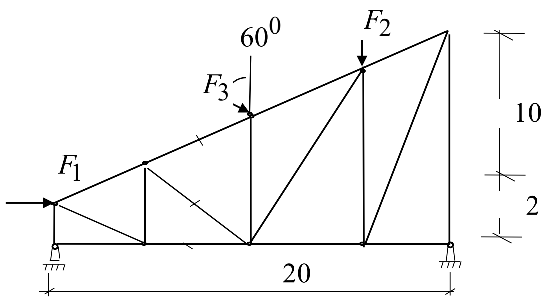
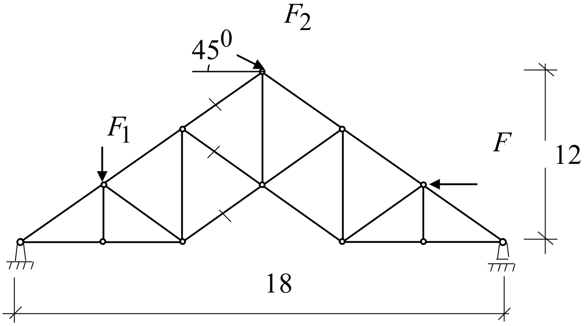
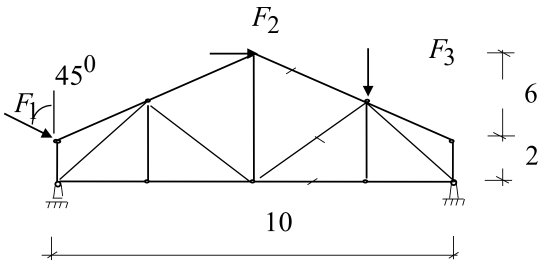
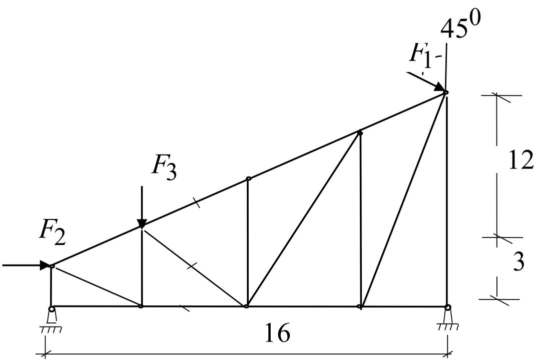
7.	$q_{сн} = 0{,}7$ кН/м^2, $a = 0{,}4$ м. 
8.	$q_{сн} = 0{,}6$ кН/м^2, $a = 0{,}2$ м. 
9.	$q_{сн} = 0{,}4$ кН/м^2, $a = 0{,}2$ м. 

10.	$q_{сн} = 0{,}4$ кН/м², $a = 0{,}4$ м. 
11.	$q_{сн} = 0{,}7$ кН/м², $a = 0{,}4$ м. 
12.	$q_{сн} = 0{,}8$ кН/м², $a = 0{,}5$ м. 

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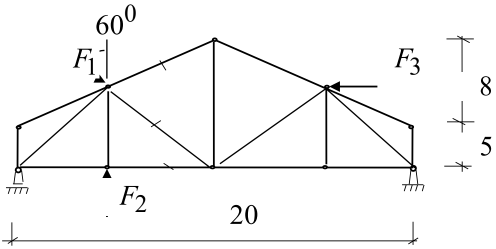

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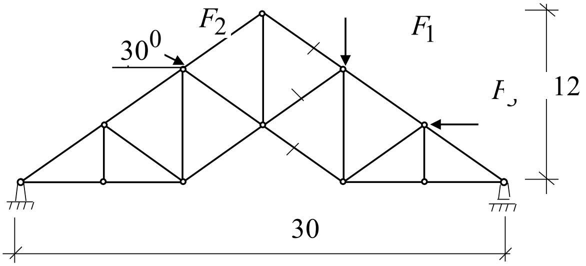

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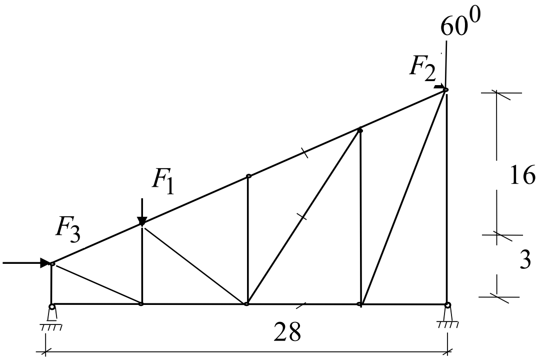

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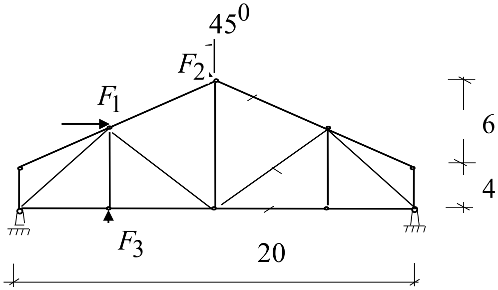

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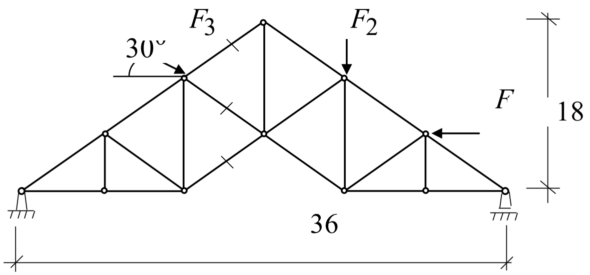

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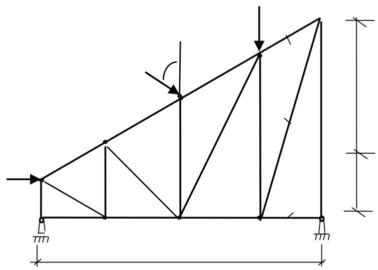
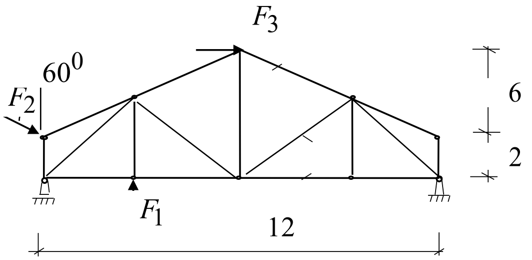
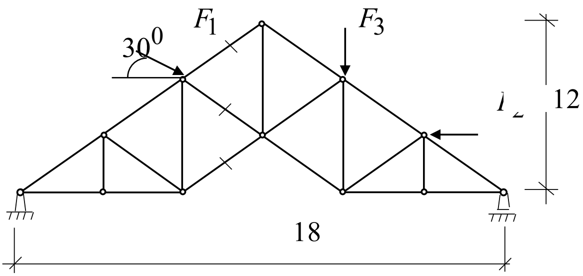


19.	$q_{сн} = 0{,}7$ кН/м^2, $a = 0{,}4$ м. 
20.	$q_{сн} = 0{,}6$ кН/м^2, $d = 02$ м. 

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A. KUTSENKO

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The calculations of complex beams and trusses

Secondary edition

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